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Orientations of Thirteen Apollo´s Temples: A Gnomonic Perspective

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ABSTRACT

Apollo, the Sun God, is one of the most prominent deities in the Ancient Greek religion. The temples and the oracles dedicated to the cult of Apollo correlate the selection of their geographical site with their special orientation, as based on ancient astronomical practices of symbolic and ritual importance. By studying the Ancient Greek temples, as for example the temples of Apollo, various researchers in the field of Archaeoastronomy, as for example, Ranieri discovered their special, non-random orientation. In this paper the special orientations of thirteen temples of Apollo are studied by the application of a novel criterion, the criterion of the platonic gnomonic factor (fgp). The majority of the temples have special arithmetic values of their corresponding fgp, and this result leads us to suggest a hypothetical application of a methodology, relating to the value of the gnomonic factor and the orientation of the temple; implemented at the time of the definition of its construction. We find that for six out of thirteen temples of Apollo, a correlation between the gnomonic factors implied by the site selected for them, their specific orientation as well as the size of their basement, exists. This analysis of the sites and orientations from a gnomonic perspective could shed light to some cultural aspects of the ancient civilizations, and their symbolic relation with the cult of this Sun God..

Keywords: *platonic gnomonic factor, summer solstice, pythagorean triads, oracles*

INTRODUCTION

A couple of years ago, a broad study of ancient temples and oracles in Greece was published by M. Ranieri in 2014 [1]. In this interesting work, Ranieri describes a very special behaviour of the orientation for the majority of the studied temples, by identifying the diagonals of the temples to be oriented, mainly, towards one of the cardinal points. Furthermore, he establishes that the dimensions of each temple obeys a condition related to a Pythagorean triangle, as already identified in a previous study. [2]

From these results, the main axis orientation of the temples fall in an apparent semi-arbitrary behaviour, confirming the assertion made few years before by Boutsikas [3], where she says: "[The] analysis demonstrates that there is no evidence to suggest a distinction in the orientation of the temples of … [semi-gods] deities towards the west. In the … [olympic] deities there is indeed a preference to the east, but the same preference exists also (to a lesser degree) for the… [semi-gods worship] temples... The general data analysis leads once more to the conclusion that we need to look at individual cults and temples".

However, while studying a sample of thirteen temples dedicated to Apollo, the Ancient Greek Solar deity, a special feature was found for the diagonals of the temples: they appear to be oriented towards one of the cardinal points. From this sample of the temples, eight temples' diagonals point towards the East; three of them point towards South; two to the North; and, none to the West (See Table 1). Castro and Liritzis in [4, 5], have argued that the scholars from 19th century assumed that the orientations of the buildings in Antiquity were mostly associated to celestial bodies and to the sunrise at one of the four solar stands, that's is the two equinoxes and the two solstices.

Many distinguished researchers [6, 7, 8] enrich with their archaeoastronomical results the analysis of archaeological evidence, about the cult of gods and the related rituals. In the case of the Ancient Greek temples, for example the temples in Delphi, in Delos, and other ritual places of central cultural and religious importance, basic results have been obtained, based on literary and worship, as well as on working field evidence [9, 10]. These results lead to a better understanding of the culture and reconstruction of some religious practices and cosmological beliefs [11].

Other recent analysis have suggested that the orientation of Greek temples could be associated with celestial phenomena: Aurorae as in Bassae temple [12]; Dolphin constellation [6], with this field of study producing already significant archaeoastronomical and archaeological results [13, 14].

In our study, we have made emphasis on the worship of Apollo as a Sun God. In ancient times, the observation of the Sun was intimately linked with the Sun deities [15]. In many ancient civilizations, including the Wessex tradition of Stonehenge, the Greek Minoan and Cycladic civilization, the Egyptian civilization, the Phoenician civilization, the civilizations of the Middle East and the Near East, the Mesoamerican civilizations [16], and other civilizations, all of them included within their religious practices Sun gods and Sun deities, as well as Lunar Gods and Lunar Deities. In these civilizations the determination of the solstices is immediately related to the cult of a Sun god [17, 18], since these days play a great importance for all ancient agricultural and hunting and gathering civilizations, already from the era of the Neolithic revolution [19].

In Methodology section, we introduce the notion of the gnomonic factor and the platonic gnomonic factor. We analyse the location and the orientation of a characteristic sample of the thirteen temples of Apollo, which are of most religious and historical importance for the Ancient Greek and Greco-Roman world. The distribution of their locations spreads throughout the geographical uterus of the Mediterranean civilizations. This sample of temples applies to the main features of Ranieri's results [1, 2], as it is shown in Table 1.

In Section 3, we analyse this chosen group of thirteen temples by associating their corresponding Pythagorean triangles and their Pythagorean triads (P-Triads), as discussed in [20] (see table 1).

In Section 4, we introduce the main idea of the paper: the orientation of the foundations (basements) of the temples could be given by the measurement of the shadow of a gnomon at the specific moment when the Sun casts a shadow of length twice the length of the gnomon. This corresponds to the case when the platonic gnomonic factor would equal to unity. This does not happen during noon or at culmination but at a moment earlier or later it.

Finally, in Discussion section, we suggest a possible procedure for defining the specific orientation at design time of the Apollo's temples during their first foundation, at the time of the determination of the alignment and the size of their basement, and its possible historical and cultural origin. A Conclusion section at the end closes the paper.

Sun and Deities

In the ancient Greek civilization two main deities relate their existence with the Sun, the God Helios [21] and the God Apollo [22]. God Helios, a Titan, counts among the important Sun related deities within the ancient Greek Pantheon. Helios was considered to be the son of the Titan Hyperion and the Titaness Theia, and brother of the goddess Selene (Moon) and Eos (Dawn), where Helios appears in many cultural depictions as crowned with the aureole of the Sun. Helios drives the chariot of the Sun with four steeds across the sky each day, defining the power and the regularity of Sun's motion on the celestial sphere [23].

In Figure 1 Helios drives his chariot in full glory over the sky. We emphasize that Helios appears depicted not at sunrise or sunset, but in the middle of the day, shown in its full glory. This observation plays a crucial role in our study.

Figure1. *Depiction of the Sun god Helios, with the aureole of the Sun, riding a quadriga of four winged steeds. (Attic red figure, side A of a calyx crater, dated in the High Classical Period, ca. 430 B.C. Collection of the London British Museum, London E 466, Trus-tees of the British Museum).*

After the Titanomachy, the ten-year series of battles among the Titans and the Olympian Gods, the Titans were overthrown by the Olympians, with Zeus, King of all Gods, as the leader. Son of Zeus and Titaness Leto, Apollo had an elder twin whose name was Artemis [24].God Apollo becomes identified with God Helios, and replaces his existence after the Titanomachy, while the cult of Apollo replaces the cult of Helios, after embracing all of the symbolisms and attributes of the former deity.

Apollo and the Sun

Apollo eventually becomes one of the most important Gods of the ancient Greek and Roman world. The cult of Apollo spreads throughout the Mediterranean, and is accompanied by the evolution of the Greek architecture, which is based on the rightness of form and on the usage of mathematical rules, in the form of arithmetic and geometric symbolic proportions, especially through the flourishing doctrine of Pythagoreanism [25].

Apollo was considered to have a special gift received from Zeus, that of prophecy. Thus, many important temples and oracles were dedicated to his cult, counting to the most important religious and cultural centres of the ancient Mediterranean [23]. These are the reasons we choose to study a characteristic sample of important ancient Greek temples dedicated to Apollo.

The concept of the physical law-based regularity of the motion of the Sun is of central importance in the Ancient Greek religion, while this tradition is transfused to the Ancient Greek philosophy and astronomy. This is emphasized in the centrality Logos plays for all physical phenomena and procedures [26]. This is stated in the famous quote of Heraclitus: “The Sun will not overstep his measures; if he does, the Erinyes, the handmaids of Justice, will find him out” [27]. This refers to the assigned measures of the spatial and temporal motion of the Sun in the sky, on his daily and annual path, passing from its position at the Winter Solstice to the position it occupies at the equinoxes, and then to the position of the Summer Solstice. This ancient Greek mentality could provide a crucial hint for the orientation procedure of the Ancient Greek temples followed by the architects and the skill full technicians of the Ancient Greek and Greco-Roman world.

Vitruvius, a representative of this long tradition, in his book De Architectura (Ten Books on Architecture), and especially in Chapter V, with the eloquent title “How the Temple should Face”, states: “… the temple and the statue which is in the shrine look towards the western quarter of the sky…” [28]. Here, we recognize that the temples are aligned towards a particular region of the sky.

Temple’s Orientation

The orientation of the temples towards East and towards the day of birth of the Gods is also an important factor, and this key idea is supported by Ranieri’s studies [1, 2]. In these studies it is shown that the temples were principally oriented in such a way, so that the diagonal of their first foundation was oriented towards a cardinal point. We suggest that the alignment of the studied temples of Apollo is based on the observation of the Sun on the days of the main stands (winter and summer solstices), when its position is high up in the sky, as also symbolically depicted in the Attic vase of Figure 1, and not during sunrise or sunset. For this reason we use a concept presented several year ago by one of us: the platonic gnomonic factor [29].

We also take in consideration that the possible symbolic and ritual significance of this gnomonic factor could shed light to the enigmatic quote from Heraclitus: “[Concerning the size of the sun: it is] the width of a human foot” [30]; which can be interpreted as a measure of the Sun’s image produced by a ‘pinhole’.” [18].

METHODOLOGY

We assume that the temporal determination of the winter and the summer Solstices, and the length of the shadow cast by the Sun from a gnomon at noon during the days of the solstices, was of special social, astronomical and ritual importance. This importance attributes symbolic and mystical meaning to the days of the solstices. For example, the length of the cast shadows by a gnomon plays an important role, even today, for the determination of the Islamic times of prayer [31].

In consequence, the methodology used has its roots on gnomonic and, specially, in the gnomonic factors, introduced few years ago by one of us (RPE) as has been already stated.

Table1. *Selected Temples of Apollo w/Pythagorean Triads*

Temple or Oracle	Squaring Triad	Type[2]	Axis[3] (°)	Diag[3](°)
Delphi, Sanct. of Apollo, Old Temple of Apollo	2 x D=3-4-5 (x 22)	SHORT	36.7	0.0
Thermon, Temple of Apollo	G=12-35-37(x 15/4)	DIAG	252.3	271.2
Athens, Agora, Temple of Apollo Patros	Φ=21-34-40(x 3 4/21)	LONG	346.2	3.4
Claros, Temple of the Sanctuary of Apollo	M=8-15-17 (x 10)	LONG	344.0	358.9
Didyme, Temple of Apollo	2 x V=20-21-29 (x 7)	DIAG	27.1	1.6
Kos, Sanctuary of Apollo	M=8-15-17 (x 17)	DIAG	27.1	359.0
Cyrene, Temple of Apollo	WB=10-25-27 (x 3)	DIAG	342.1	3.9
Delphi, Sanctuary of Apollo, Tes. XVII	76-114-137 (x 1/4)	DIAG	303.9	270.3
Bassae, Newer Temple of Apollo	W=5-12-13 (x 20)	DIAG	87.4	93.7
Delos, Apollo´s Great Temple	MB=12-22-25 (x 2)	LONG	282.8	267.5
Corinth, Temple of Apollo	W=5-12-13 (x 15)	DIAG	21.8	0.0
Eretria, Temple of Apollo	WA=10-23-25 (x 7)	SHORT	321.7	2.7
Dreros, Temple of Apollo Delphinios	π/2=7-11-13 (x 7/2)	LONG	71.6	89.3

[1] From Ranieri's Report [2]. [2] Type of design of the temple: DIAG is rectangular; LONG is defined as half the axis; SHORT is defined as cutted by the middle. [3] Angle measured from East direction.

Background

The selection of the sites and the orientation of the temples of the Ancient Greek and Greco-Roman epoch satisfied both astronomical criteria, and the oracular and visionary insights of the temple's priests and pythonesses functions, that is geographical locations with specifically geological characteristics, such as the place of a fountain or a place of exhalation of estrange gases [32].

Our proposal is that, together with these relevant criteria, there could have been applied some kind of gnomonic and/or platonic gnomonic factor at the time of the determination of the sites of Apollo's temples, as well as their special orientation and architectural design. A geometrical and symbolic meaning of these solstice shadows could be quantified arithmetically by the use of the gnomonic factor, as well as of the platonic gnomonic factor. Within the culture of Stonehenge, the presence of the gnomonic factor becomes apparent [33], and the practice of the pre-scientific gnomonics can be traced in the architectural design of Stonehenge. The application of the gnomonic and the platonic gnomonic factor involves the observation of the Sun at noon, a special position on the celestial sphere. Also, the azimuth and the elevation of the Sun above the horizon at the solstice days is implied by these special shadows.

Definition of the Gnomonic Factors Fg and Fgp

The *gnomonic factors* are defined by the use of a right triangle, the gnomonic triangle, whose perpendicular sides are formed by the height of the gnomon and the shadow cast by the Sun.

The first step of our study was to locate the temples using Google Earth and other geographical tools [34, 35]. Then, the shadows of a gnomon of one unit length are obtained for the two Solstices by the use of Equation (1):

$$XSs = \frac{1}{\tan(90° - \delta \mp \phi)} \quad (1)$$

Here, $X = W$ denotes the Winter Solstice and the (-) sign in the Equation is used; and, $X = S$ denotes the Summer Solstice and the (+) sign in the Equation is used. The symbol δ denotes the Latitude of the chosen geographical location, and ϕ is the obliquity of the ecliptic at the date considered. These two shadows allow us to define the *gnomonic factor* as their difference [29]:

$$fg = WSs - SSs \quad (2)$$

and the *platonic gnomonic factor* when $\delta + \phi = 45°$:

$$fgp = WSs - 1 \quad (3)$$

In Table 2, we list the corresponding values for the thirteen sites calculated at noon time. For the obliquity of the ecliptic we have used the Laskar formula [36] and also considered the possible dates of the foundation of the temples, which also appear in the table, as given by historical sources and studies [37].

We observe that the gnomonic factors acquire significant values for the prestige cases of the temple of Apollo at Delphi and the temple of Apollo at Thermon.

Table2. *Gnomonic Factors of Apollo's Temples at Noon*

Temple or Oracle	Lat (N)	Date[1] (BC)	*fg* [2]	*fgp*[2]
Delphi, Sanct. of Apollo, Old Temple of Apollo	38° 28' 56.33"	ca. 525	1 2/3	1
Thermon, Temple of Apollo	38° 34' 44.36"	ca. 630	1 2/3	1
Athens, Agora, Temple of Apollo Patros	37° 58' 19.24"	ca. 338	1 3/5	6/7
Claros, Temple of the Sanctuary of Apollo	38° 00′ 18.03″	ca. 300	1 3/5	6/7
Didyme, Temple of Apollo	37° 23' 05.41"	ca. 540	1 5/9	4/5
Kos, Sanctuary of Apollo	36° 52' 32.58"		1 5/9	7/9
Cyrene, Temple of Apollo	32° 49' 24.60"		1 1/3	½
Delphi, Sanctuary of Apollo, Tes. XVII	38° 28' 56.28"	ca. 360	1 2/3	1
Bassae, newer Temple of Apollo	37° 25' 46.99"	ca. 450	1 4/7	4/5
Delos, Apollo´s Great Temple	37° 24' 01.80"	ca. 350	1 4/7	4/5
Corinth, Temple of Apollo	37° 54' 21.71"	ca. 600	1 3/5	6/7
Eretria, Temple of Apollo	38° 23' 51.76"		1 2/3	8/9
Dreros, Temple of Apollo Delphinios	35° 15' 23.11"		1 1/2	2/3

[1] As reported by Liritzis & Vassiliou [5]. [2] Calculated with shadows observed at noon time.

For the gnomonic factors defining the foundations orientation the values were calculated using Eq. (2) and Eq. (3), but the corresponding values of the shadow were obtained from the Sun's elevation as follows:

$$WSs = \cos(Se) \tag{4}$$

and

$$Se = Sun\acute{}s\ elevation\,(Az). \tag{5}$$

Here, *Se* denotes the elevation of the Sun at the azimuth *(Az)* implied by the Temple's axis. This elevation was obtained with the Sun's position simulator program Stellarium [38]. These gnomonic factors are listed as primed gnomonic factors, and are listed in Table 3, in order to distinguish them from the gnomonic factors listed in Table 2.

Gnomonics and the Astronomical Traditions

The use of gnomons as an astronomical instrument appeared in the ancient cultures at different stages of their development. Here we only recover those aspects related to the use of shadows cast by a gnomon to contextualize the Greek implicit knowledge that will be used in the analysis.

The Use of Shadows in the Arabic and Islamic Cultures

We trace back in historical time for finding evidence about the use of the calculus of shadows cast by a gnomon, and the corresponding use of the gnomonic triangle. The reminiscence of the calculus of shadows can be traced both in the Indian, as well as in the Arabic and Islamic mathematical traditions. Within the context of the trigonometric functions, tangent and cotangent are defined and measured via the (linear) shadow cast by a gnomon [39] [Joseph, 2000].

Al-Battani (Albategnius in Latin), (c. 858-929), one of the most prominent figures of the Arabic and Islamic tradition, a great mathematician and astronomer [40], uses extensively the calculus of shadows in astronomy and trigonometry, by studying the vertical and the horizontal gnomons, especially in his monumental work Kitab al-Zij ("On ascensions of the signs of the zodiac") [41]. Al-Battani introduces the horizontal shadow (*umbra extensa* in Latin translation), as well as vertical shadow (*umbra versa*) [42], that is, the cotangent and the tangent trigonometric function [43]. Al-Battani gives the rule for the determination of the elevation of the Sun above the horizon in terms of the length s of the shadow, cast by a vertical gnomon of height h, continuing the calculus of shadows found in the gnomonic and platonic gnomonic factor.

His rule, $s = h \sin(90° - \theta)/\sin\theta \approx s = h \cot\theta$.

Al-Battani completes his efforts by calculating a table of shadows, that is a table of cotangents, for each degree from 1° to 90° [44]. Also, through his work, Al-Battani also uses the Indian half-chord function, that is the sine function, while his use of the trigonometric function of the sine spreads out in Europe by the transmission of his astronomical work [39]. We emphasize the fact that the measurement of sine function gives in a direct and straightforward manner the North-South direction of a geographical points at the place. Al-Battani also uses the algebraic method of manipulating mathematical operations, instead of the then preferred Greek geometrical method encountered in the European tradition [45, 46]. He might be considered as the master of the calculus of shadows as was considered in Marie´s book, cited above.

Another prominent figure of the same tradition is the famous scholar Al-Biruni [47], with his most important work, titled as the Exhaustive Treatise on Shadows (‘*Ifrad al-maqal fi amr al-zilal’*) sheds also light into this important branch of human knowledge and practice. In this monumental work Al-Biruni studies the science of Gnomonics, the history of the tangent and the secant functions, the applications of the calculus of shadows to the astrolabe and other astronomical instruments, the shadow observation for the solution of many astronomical problems, and the shadow-determined times of the Muslim prayers.

The Arabic and Islamic astronomical and mathematical tradition was widely influenced by the Greek, the Indian and the Chinese astronomical traditions, and fertilized the European astronomical and mathematical tradition, through the transmission of fundamental works of great scholars into the European continent.

The Gnomonics in the Indian and Chinese Culture

It is costumed to classify the Indian astronomy into three broad periods, that is the Vedic astronomy (c. 1200-400 B.C.), the post-Vedic astronomy (c. 400 B.C. to 400 A.D.), and the Classical and Medieval astronomy (from A.D. 400 onwards). Indian astronomy is treated as an entity which undergoes phases of growth and phases of accretion of ideas, indigenous and foreign, within a process of cultural diffusion [48]. The determination of the important ritual dates and civil harvest festivals depended strongly on the determination of the solstices and equinoxes, thus on the use of the calculus of shadows cast by a gnomon. This tradition passes over to the Vedic astronomy, where the trigonometric calculations of tables of sines and cosines in intimately related with the position of the Sun in the sky, and the casted gnomonic shadows.

In the important Chinese astronomical and mathematical treatise, the *Zhou Bi Suanjing* we encounter all the basic applications of the calculus of shadows, such as the determination of the North and South direction of a given place. Also, the remotely measurement of distance of objects or lengths comes from the use of the circle and the try square; i.e., the right triangle. This method was accomplished by the use of upright staffs or posts as gnomons [49]; this could have been one of the firsts measurements of shadows in human history. According to the belief of the mathematicians following the Chinese mathematical tradition, the origin of mathematics is derived by the measurement of the right triangle and the circle [50].

The Calculus of Shadows in the Greek Culture

In the Greek astronomical and mathematical tradition a new form of geometry and astronomy was developed during the Ionian Renaissance [51]. Here, we encounter the introduction and use of the scientific discipline of Spherical Trigonometry, for the solution of various positional astronomical, geographical and cartographic, as well as astrological problems [52].

Also, the autonomous branch of the science of Gnomonics (the science of the Horology of the Sundials) plays a most significant role in the Greek culture [53]. The development of the design and of the construction of numerous different basic types of sundials, both fixed and portable, flourishes, accompanied together with the introduction of numerous astronomical instruments, such as the astrolabe. During the Hellenistic epoch the gnomonic and the stereographic projection, together with the study of the conic sections, were introduced and studied extensively [54]. We see that this epoch of a scientific revolution bears among its fruits the Antikythera Mechanism [55, 56, 57], the astrolabe, etc.; an exhaustive list of observational astronomical instruments and sundials of all types could be found in [58].

Thales (ca. 624-546 B.C.), the first mathematician, measures the height of a pyramid by the use of a gnomon. Anaximander (ca. 610-546 B.C.) uses the gnomon on a flat surface as a clock by drawing three circles around the stick; about each two hours the shadow of the Sun passed from one circle to the other. He also uses the gnomon for determining the meridian by marking the exact places where the shadow crossed each circle. Thus, he deduces that the bisecting line was common to the three circles for the same day; north-south direction. The solstice where found from the measurement of the lengths of the shadows while the Sun was at this meridian line, at noon time, with the longest shadow for winter the shortest for summer [59]. The gnomon is also used for the determination of the meridian line and the epochs of solstices; also, it was a basic tool for measuring the inclination of the equator above the horizon and to know the obliquity of the ecliptic [42].

The Orientation of Apollo’s Temples

From a previous study, Perez-Enriquez [29] has found that the site of Dodona, the first oracle in Greece according to Herodotus [60] [450 B.C.], dedicated to Zeus, the king of the Gods of Pantheon, could have been chosen because its location satisfies an important gnomonic factor criterion, that is, its platonic gnomonic factor *(fgp)* is equal to unity (equal to 1/1). Could Dodona define a trend?

Locations of Oracles and Temples

As it can be seen in Table 2, Delphi has also a platonic gnomonic factor (fgp), whose value is equal to 9/10, close to one. Delphi is also considered to be the most important place of worship of God Apollo, and of equal importance of the oracle of Zeus in Dodona, thus the two selected sites have a similar important gnomonic criterion.

For each one of the thirteen temples, studied here, we determine its geographical location (latitude and longitude) and the orientation given by the main or small axis of the basement of the temple. Then, we apply a Pythagorean Triple the direction of the diagonal of the basement of the temple, as reported in [2]. With the aid of the latitude and the implied azimuth, we recalculated the corresponding gnomonic factors, for each case of our sample.

All of the selected temples represent a set of important worship centres of Apollo, of equal religious status with the Dodona and Delphi cases, and are listed in Table 3. For example, Delphi, the Panhellenic sanctuary of Pythian Apollo, was regarded as the centre of the world, called as the omphalos (the navel of the world). Its recognition and fame extended throughout the great ancient civilizations of the Mediterranean [Delphi, Temple of Apollo]. Also, the temple of Apollo at Thermon marked the meeting place of the Aetolian League, where annual festivals were held; the temple of Apollo Patros in Athens belongs to the Hellenistic period, where Athens becomes the centre of a cosmopolitan institution for the ancient Greco-Roman world, and one of the leading centres for the study of philosophy (including Plato's Academy, Aristotle's Peripatos, the Stoa of the Stoic school of philosophy, and the Kepos, the Epicurean school of philosophy) [61].

Their Platonic Gnomonic Factors

Since we obtained the corresponding gnomonic factors for the sample of the thirteen temples of Apollo, as listed in Table 2, we calculate the same gnomonic factors, by the use of Eq. (2) and Eq. (3), in the case the Sun is observed when having an elevation nearly equal to the noon time elevation, as defined by either the main axis, or the short axis of the basement of the temple, during the Winter and Summer solstice, and at the hours defined by the corresponding azimuths of either the main or the short axis (Eq. (4) and Eq. (5)). The orientations obtained in Ranieri's study give the main axis of the temple taken from the East direction. We then considered the direction towards which the shadow of a gnomon goes along one of the temple's axis, and obtain the *new gnomonic factors*. The values of the new *gnomonic factors* are denoted by the primed symbol *fg'* and by *fgp',* and are listed in Table 3, and corresponding to new azimuths, as we define them in our study. The diagrams where drawn using the data specifying type of triangle and axis direction as given in Table 1.

Table3. *Gnomonic Factors of Apollo's Temples at Noon*

Temple or Oracle	Lat (N)	Axis[1]	Diag[1]	Az[2]	*fg'*[3]	*fgp'*[3]
Delphi, Sanct. of Apollo, Old Temple of Apollo	38° 28' 56.33"	36.7	0.0	143.3	2 5/7	2
Thermon, Temple of Apollo	38° 34' 44.36"	252.3	271.2	197.7	1 4/5	1
Athens, Agora, Temple of Apollo Patros	37° 58' 19.24"	346.2	3.4	166.2	1 5/7	1
Claros, Temple of the Sanctuary of Apollo	38° 00′ 18.03″	344.0	358.9	164.0	1 3/4	1
Didyme, Temple of Apollo	37° 23' 05.41"	27.1	1.6	152.9	2	1 1/4
Kos, Sanctuary of Apollo	36° 52' 32.58"	27.1	359.0	152.9	2	1 2/9
Cyrene, Temple of Apollo	32° 49' 24.60"	342.1	3.9	197.9	1 1/2	0 2/3
Delphi, Sanctuary of Apollo, Tes. XVII	38° 28' 56.28"	303.9	270.3	146.1	2 3/7	1 5/7
Bassae, newer Temple of Apollo[4]	37° 25' 46.69"	91.1	103.9	178.9	1 4/7	0 4/5
Delos, Apollo´s Great Temple	37° 24' 01.80"	282.8	267.5	167.2	1 2/3	0 8/9
Corinth, Temple of Apollo	37° 54' 21.71"	21.8	0.0	201.8	1 6/7	1 1/8
Eretria, Temple of Apollo	38° 23' 51.76"	321.7	2.7	141.7	4 3/4	4
Dreros, Temple of Apollo Delphinios	35° 15' 23.11"	71.6	89.3	161.6	1 3/5	0 5/6

[1] Orientation taken from East to North [Ranieri, 2014].

[2] Azimuths for observation of the Sun; the elevations obtained with *Stellarium* [Chereau et al., 2015].

[3] Gnomonic Factors obtained at the corresponding azimuths.

[4] For this temple we made a correction of 10° in both the Axis and the Diag (Moussas' observation), (Moussas, 2016, personal communication).

Therefore, the Sun's elevations, as obtained with the aid of *Stellarium* [38], were calculated for the specified dates, as presented in Table 3, where the platonic gnomonic factor (*fgp'*) assumes a significant value. The angles discussed in this Table also appear in Figure 2. Here, a schematic depiction of a temple has its axis and diagonal identified, together with the direction of the azimuth of the Sun.

At least eight out of the 13 sites obtain a meaningful gnomonic factor value. In the case of the Thermon, the Athens, and the Claros temples, we have that fgp' = 1. This means that the position of the Sun at the sky, the length of the gnomon and the cast shadow form a Plato's Triangle, with a Sun's elevation around to 26.9° above the horizontal and at Winter Solstice. The Delphi, the Didyme and the Kos temples obtain a gnomonic factor value equal to 2; while the Eretria temple is characterized by a platonic gnomonic factor value equal to 4.

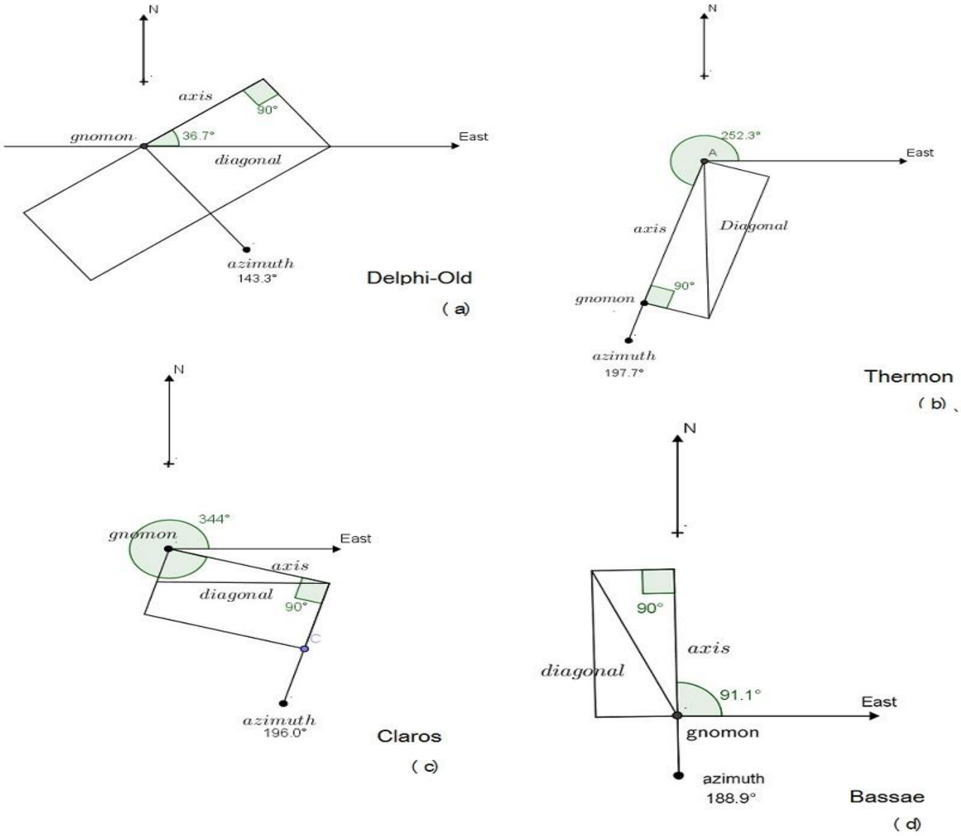

Figure2. *Diagrams of Apollo's temples at: a) Delphi; b) Thermon; c) Claros; and, d) Bassae newer, temples.*

The Corinth and the Dreros temples, we obtained an almost integer value for their fgp' gnomonic factors. Here, the Summer solstice shadow at the complementary orientation (248.4° and 108.4° of azimuth, respectively) reaches the value of the one-half (½) of the gnomon. This means that again their gnomonic triangles correspond to Plato's Triangles. There seems to be a slight variation from this suggested principle in the case of the Delphi temple, the Bassae New temple, and the Delos temple, with the corresponding values being equal to 2/3, 4/5 and 2/5, respectively. In the case of Bassae New temple, and since this temple was erected for Apollo Epicurios, as an acknowledgement for the end of the plague affecting Philae, according to the historical sources, we argue that this temple was not dedicated to Apollo as a Deity of the Sun, but as a Deity of Healing.

DISCUSSION

The number of cases showing significant values of the primed platonic gnomonic factor (*fgp'*) could suggest that a gnomonic observation procedure was followed while defining the orientation and the sizes of the foundations of the temples of Apollo, by the architects or the skill full technicians, possibly the harpedonaptai, or rope-stretchers [62] [Gordon, 1969].

The first steps could have involve the position of the Sun in the sky in a selected date (winter solstice), when the Sun projected a shadow equal to one or twice the gnomon length (a special *fgp* value), and the subsequent steps refer to the selection of a Pythagorean triple, in order to accommodate the diagonal of the temple towards one of the cardinal points, preferably towards the East direction. This choose could have been made with the aid of a knotted rope. Today, we can apply some backward engineering for proposing a possible procedure used by the architects while determining the foundations of the temples, at the design stage of the building.

The possible steps of this procedure include:

- After the selection of the site, a horizontal surface is prepared, with a vertical gnomon placed at its centre;
- Two concentric circles are drawn, with the gnomon as their centre, one of them of radius equal to the length of the gnomon, and the other with radius twice as long the length of the gnomon;
- At Winter Solstice, the architect or the rope-stretcher waits until the shadow of the gnomon crosses both circles (according to this method, it is also possible to determine the azimuth when, for example, the *fgp'* reaches the value one*)*;
- Using the knotted rope (the *harpedon* used by the *haprpedonaptai*) the line from the gnomon to the crossed points at the circles is drawn, and the perpendicular direction is determined as the arithmetic rope is stretched in such a way, so that it forms a right triangle. The number of the knots of the arithmetic rope also determine a specific Pythagorean Triad. Then;
- The selected Pythagorean Triad is used for aligning the foundations of the temple, as the diagonal of the arithmetic rope points towards a cardinal point, for example due East;
- The Pythagorean Triangle can be scaled until the desired dimensions of the building are reached.

The circles of this proposed method are used for obtaining the desired value of the gnomonic factors. This procedure is analogous to Anaximander's procedure for the measurement of time [63].

The gnomonic factors obtained for the specific azimuth of the Sun, combined with the orientation of the diagonal of the building, could have served for the determination of the extent of the temple's basement, and its orthogonal design. The determination of the azimuth of the Sun is strongly correlated with the symbolic, religious and ritual role of the Sun God Apollo. After this first stage of our proposed method, the determination of the two sides of the orthogonal shape of the basement is acquired, by considering its diagonal as belonging to a specific Pythagorean Triple (that is, to a Pythagorean triangle). The diagonal is pointing towards one of the cardinal points, while the arithmetic values of the two perpendicular sides are defined by the corresponding arithmetic values of the numbers of the Pythagorean Triple.

As it is stated in the Surya Siddhanta after nine questions about the earth, the Sun and their dimensions: "...10. Having heard the speech thus addressed by MAYA with his best respects, the man ... related to him the secret Second Part of the work. 11. O MAYA, hear attentively the secret knowledge called *Adhyatman* (or means of apprehension) which shall tell you..." [64].

The pre-Vedic astronomy, as well as all astronomical traditions of the prehistoric and historic civilizations in Mesoamerica [16], in Europe, in Africa and Asia, might encompass the theory and the practice of the *gnomonic triangle*, and the accompanying *gnomonic factors*. Our suggested *platonic gnomonic factors*, and this long-standing tradition, could be integrated within the advances of the great mathematicians of the Greek [65], the Indian, the Chinese, the Arabic and Islamic and the European traditions. The long tradition of the harpedonaptai, coming from earlier Egyptian times, could serve as another direction for investigating the history and the role of the *calculus of shadows*. This tradition might belong to our World Heritage.

CONCLUSION

The gnomon could have been used in the ancient prehistoric and historic civilizations. The observation with the gnomonic triangle could have a religious significance, and also attribute a symbolic meaning to geographical places, where specific arithmetic values of the Sun's cast shadows were obtained. These values could refer to a symbolic "sacred arithmology" associated with the *calculus of the shadows*, while this practice could have been incorporated in the "secret teachings" of the class of the architects, the skill full technicians, such as the rope-stretchers or harpedonaptai, and the priesthood of these civilizations, who transmitted this knowledge. Apollo's oracles and temples would have been oriented according the position of the Sun in the sky, and at locations previously selected by mystical, and gnomonic criteria.

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