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# Fuzzy Logic and Mathematical Education

**Angel Garrido**

*Department of Fundamental Mathematics, Faculty of Sciences UNED, Madrid*

***Corresponding Author:** Angel Garrido, Department of Fundamental Mathematics, Faculty of Sciences UNED, Madrid*

## ABSTRACT

*From remote times, the history of the human being is developed by a successive chain of steps and sometimes jumps, until the relative sophistication of the modern brain and its culture. The historical origin of the Artificial Intelligence is usually established at Darmouth Conference (1956). In this year, John McCarthy coined the term, and defines it as "the science and engineering of making intelligent machines". But it is many more, multiply connected with many other fields, as Neuroscience or Philosophy. And we can find many more arcane origins, perhaps to Plato, Raimon Llull, Leibniz, Pascal, Babbage, Torres Quevedo, ..., with its attempts to create thinking machines. Also the construction of artifacts devoted to more quick, and based in logic computation. So, A I will be interpreted as the area of Computer Science focusing on creating machines that can engage on behaviors that humans consider intelligent. Recall topics as Turing test, Strong vs Weak AI, Chinese Room argument, and so on. Until now, more than fifty years of modern AI. Researchers are attempting to create systems which mimic human thought, or understand speech, or beat the best human chess player. Understanding intelligence and creating intelligent artifacts are the twin goals of AI. Also, we can consider, in more recent times, very great mathematical minds, as von Neumann, Wiener, Shannon, Turing, Moisil, or Zadeh.*

**Keywords:** *Mathematical Education; Mathematical Logic; Fuzzy Logic; AI; Computer Science.*

## ON ARTIFICIAL INTELLIGENCE

Frequently, *AI requires Logic*. But Classical Logics show too many insufficiencies. Hence, it has been necessary to introduce more sophisticated tools, as *Fuzzy Logic*, *Modal Logic*, *Non-Monotonic Logic*, and so on. Recall that Mathematics will be a mere instance of First-Order Predicate Calculus, and therefore, belong to applied Monotonic Logic. So, we must show the limitations of classical logic reasoning, and the advantages of Fuzzy Logic.

Among the things that *AI* needs to implement a representation are *Categories*, *Objects*, *Properties*, *Relations*, and so on. All they are connected to Mathematics, as well as being very good and illustrative examples. For instance showing Fuzzy Sets together with the usual Crisp, or Classical Sets, as a particular case of the previously mentioned Fuzzy Sets; or introducing concepts and strategies from Discrete Mathematics, as the convenient use of Graph Theory tools on many fields.

The problems in *AI* can be classified in two general types, *Search Problems* and *Representation Problems*. Then, we have *Logics*, *Rules*, *Frames*, *Nets*, as interconnected models and tools. All they are very mathematical topics.

The origin of the ideas about thinking machines, the mechanism through work the human brain, the possibility of mimic its behavior, if we produce some computational structure similar to neuron, or to neural system, their synapses or connections between neurons, to produce Neural Networks... All this can appear with resonances of a Science Fiction history, or perhaps a movie, but it is a real subject of study, and it is so from many years ago, and more in the last times.

The basic purpose of the A I will be to create an admissible model for the human knowledge. Its subject is, therefore, "pure form". We try to emulate the way of reasoning of a human brain. This must be in successive, approximating steps, but the attempts proceed always in this sense.

Initially, the work in AI was over idealizations of the real world. The fields were, therefore, "formal worlds". Such search procedures were into the Space of States, which contains the set of all states, or nodes, in the case of representation by graphs, that we can obtain when we apply all the disposable operators. Many early AI programs used the same basic algorithm. To achieve some goal (winning a game or proving a theorem), they proceeded step by step towards it (by making each time a move or a deduction) as if searching through a maze, backtracking whenever they reached a dead end. This paradigm was called "reasoning as search".

The techniques for solving problems, in AI, can be of two types, *Declarative:* it permits the description of the known aspects of the problem. It is the Heuristic Treatment; and *Procedural:* which itemizes the necessary paths to reach the solution of the problem. It is the Algorithmic Treatment.

*To pose problems is equivalent to constructs its solutions.* This requires: an agent, the system or program to execute; a set of actions, which allow us reach such objectives; and a procedure of election, which allows us to select between distinct paths to reach its solution.

We can use a series of resources approaching problems in AI, as *Logic, Rules, Associative Nets, Frames and Scripts.* The election between these methods must be based in the own characteristics of the problem and our expectations about the type of solution. In many cases, we take at a time two o more tools, as in the case of the Frame System, with participation of Rules.

The *Inference in Rule System Based* consists of establishing the certainty of some statement, from the disposable information into BF and KB. We have two methods, going forward and going backward concatenation.

The Rules shows clear advantage on Classical Logic, where the reasoning was monotonic, with inferences without contradicts pre-existing facts. While in the RBS we can delete or substitute facts of the Base of Facts, according to the new inferences. All may be provisional and modifiable. This made the Reasoning Non-Monotonic.

In the case of some applicable rules at time, which must be executed firstly? Such set of Rules constitutes, in each step, the Conflict Set (which will be dynamic, obviously). The subjacent decision problem is called Resolution of Conflicts, or also named Control of Reasoning.

## SEARCHING STRATEGIES

There exist different strategies, for to elect each time a Rule: Ordering of Rules, Control of Agendas, Criterion of Actuality, and Criterion of Specificity. The Criterion of Specificity leads to execute, firstly, the more specific Rules, i.e. those with more facts in its antecedent. So, between $R_1$: if a, then b, and $R_2$: if a and d then c, we must to select $R_2$, because it is more specific than $R_1$.

We also have Mechanisms of Control in RBS. So, by Mechanism of Refractarity, by which we forbid to execute newly a Rule, once utilized, if do not exist more information which allow or recommend such case; Rule Sets, it allows us to activate or neutralize Blocks of Rules; and Meta-Rules, or Rules which treat (or reasoning) about other Rules. Such Meta-Rules can collaborate to Control of Reasoning, with the change or assignation of priorities to different Rules, according to the evolution of circumstances.

*Nets:* Between them, the more recent studies to deal with Bayesian Nets, or Networks. Before than its apparition, the purpose was to obtain useful systems for medical diagnosis, by classical statistical techniques, such as the Bayes Rule.  A *Bayesian Net* is represented as a pair (G, D), where G is a directed, acyclic and connected graph, and D will be a probability distribution, associated with random variables. Such distribution verifies the Property of Directional Separation, according to which the probability of a variable does not depends of their not descendant nodes.

The Inference in BN consists in establish on the Net, for the known variables, their values, and for the unknown variables, their respective probabilities.

The objective of BNs in Medicine is to find the probability of success with we can to give determined diagnosis, known certain symptoms. We need to work with the subsequent Hypotheses: Exclusivity, Exhaustivity and CI (Conditional Independence). According the Exclusivity, two different diagnoses cannot be right at time. With the Exhaustivity, we suppose at our disposition all the possible diagnosis. And by the CI (acronym of Conditional Independence), the discoveries found must be mutually independents, to a certain diagnosis. The usual problem with such hypotheses will be their inadequacy to the real world. For this, it will be necessary to introduce Bayesian Networks. In the searching process, we have two options: without information of the domain (Blind Search); and with information about of the domain (Heuristic Search).

In the first case, we can elect, according the type of problem, between Search in extent and

Search in depth. There are other methods, obtained from the previous, such as Searching in Progressive Depth and Bidirectional Searching, both with names sufficiently allusive to its nature. Also we can found another method, in this case not derived, the General Search in Graphs. In such procedure, it is obvious the possibility of immediate translation to matrix expression, through their incidence matrices. All these methods joined to their algorithms.

*Blind Search,* or search without information of the domain, appears with the initial attempts to solve, by idealizations of the real world, playing problems, or the obtaining of automatic proofs.

The searching process could be in state spaces. Such searching procedure has applicability on problems provided with some characteristics, as we can associate a state to each different situation of the domain; there is a series of initial states; there are some operators, which allow us the step between the successive states; and there exist a final state. In such processes, it is clear the correspondence between State and Node of the graph, and between Arc (edge or link, into the graph) and Operator.

*Searching in extent.* We advance in the graph through levels. So, we obtain the lesser cost solution, if exists. Whereas, in the *Depth Searching,* we expand one link each time, from the root - node. If we reach a blind alley into the graph, we back until the nearest node and from this, we take one ramification in the graph. It is usual establish an exploration limit, or depth limit, fixing the maximal length of the path, from the root.

*Heuristic Search,* i.e. when we are searching with knowledge of the domain. Initially, were usual to think that all the paths can be explored by the computer. But it is too optimist. Such exploration will be very difficult, because of phenomenon as "combinatorial explosion" of branching, when we expand. Its spatial and temporal complexity can advise us against its realization. For this, we need to select the more promising trajectories. In this way, we cannot obtain the best solution (optima), but an efficient approach to her.

Now, we introduce one new mathematical tool, the so called *heuristic evaluation function, f.* By such function, we assign a value, $f(n)$, to each node n. So, such $f(n)$ give us the estimation of the real distance (unknown), from the current node, $n$, until the final node, $m$.

Also, there are strategies designed for the treatment of *Searching problems with two adversaries.* In this case, the general purpose is to select the necessary steps for to win the game. Chess or Go, generally. In fact, it was its origin. For these, we may assume alternative moves. In each move, the ideal would be when the player knows his possibilities and realizes the more unfavorable move for its adversary. But is impossible control it completely, in general, because the "combinatory explosion". So, we need to develop a tree of depth searching, with depth limited. In application of the *Principle of Racionality.* For this can be desirable to dispose of a great master or a machine, as "Deep Blue". To estimate the goal, we introduce a more sophisticated function, the aforementioned heuristic evaluation function, which would measure, for each node, the possibilities of the players.

In AI, the problems can be classified according to its level. In a first level, the problems of decision, learning, perception, planning and reasoning. In a second level, the tasks of classification, representation and search. When we formulate a problem, we depart of the statement, or the explanation of it, in natural language. Fundamentally, its treatment is based on the "level of knowledge", introduced by Newell, in 1981, as "abstract level of interpretation of systems, in AI". Also is basic the "Rationality Principle", according to "if a system has the knowledge according to which one of its actions leads to one of its goals, then such action is carried out".

The problems in AI can be finally classified in two types, *Search and Representation Problems.* For this, we need concepts such as *Trees and Graphs; Structure of Facts,* for instance: piles, queues and lists; *Knowledge about the Complexity of Algorithms.* All these problems, their methods of solution or approximation tools, may be implemented in the class-room. For instance, to apply mathematical games, as *Chess* or *Go,* must improve the logical capacities of the students, in any scholar level. Because if it is in elemental level, it may be an interesting stimulant to promote hidden potentialities. And in higher levels, it may to contribute to strengthen the logical and deductive qualities of the beginner researchers. In the case of the aforementioned tools, currently very useful in Mathematics and AI, as Graph Theory tools, it may be used in explain classical and very exciting questions, as the *Halting Problem,* or the *Traveling Salesman Problem (TSP),* or the open question for the 20$^{th}$

century generations, *P vs NP*, with multiple implications from them.

Not only may be very useful into the class-room give Mathematics with support on the aforementioned Games (Chess, Checkers, Stratego, Sudoku…), but also our students can be introduced in subtle analyses, as the *Prissoner´s Dilemma*. Also it will be disposable any information about games as Chess, Go,…, its Rules, Tricks, Hints, and so, in the Web pages, being as well possible to play with them. And we can obtain information of papers, web explanations, etc., about the history of such games, which will be very illustrative and motivating for the students.

All these techniques has been implemented in the class-room with students of secondary level, increasing with them its interest in Mathematics and simultaneously, in TIC new technologies and its fundamental basis. Furthermore, with students of undergraduate university level, in studies of Mathematics and Computer Science, reaching a very positive reaction, which increments their interest and results.

By this new approach we defend, Computer Science occupies, partially and in a natural way, the role Physics and its problems have played as support of mathematical reasoning, a fact in the past two centuries (although Physics do not disappear from the view, being a necessary aid).

We propose showing such Methods through the parallel study of Mathematics and Computer Science foundations. Other Computer Science subfields could be carriers of this method too, but perhaps *AI* is the current better choice, given its characteristics, which practically coincide with many mathematical techniques and objectives.

The benefits of such an innovative educative method must consist in a more progressive adaptation of Mathematical Education to modern times, with the final purpose of producing adaptive and creative minds, capable of solving new problems and challenges.

## Fuzzy Logic and Education

We say that a Language is Formal when their syntax is precisely given. Mathematical Logic is the study of the formal languages. Usually, it is called Classical Logic, being dychotomic, or bi-valuated, only either True or False. The Fuzzy Logic is a successful generalization of the Mathematical or Classical Logic. It deals with the problem of the ambiguity in Logic. Because Classical Logic shown many insufficiencies for the problems of AI. A more flexible tool is needed, allowing for a gradation of certainty, indicating different degrees of membership to a set, or fulfillment of a property or relationship, and so on. The introduction of concepts and methods of Fuzzy Logic, where the idea of sets, relations and so on, must be modified in the sense of covering adequately the indetermination or imprecision of the real world. We define the "world" as a complete and coherent description of how things are or how they could have been. In the problems related with this "real world", which is only one of the "possible worlds", the Monotonic Logic seldom works. Such type of Logic is the classical in formal worlds, such as Mathematics. But it is necessary to provide our investigations with a mathematical construct that can express all the "grey tones", not the classical representation of real world as either black or white, either all or nothing, but as in the common and natural reasoning, through progressive gradation.

Let A and B be two fuzzy sets, not necessarily on the same universe of discourse. The implication between them is the relation $R_{A\to B}$ such that $A \to B \equiv A \times B$, where *x* is an outer matrix product using the logical operator AND. To each Fuzzy Predicate, we can associate a Fuzzy Set, defined by such property, that is, composed by the elements of the universe of discourse such that totally or partially verify such condition. So, we can prove that the class of fuzzy sets, with the operations ∪, ∩ and c (with c the complement set operation), does not constitute a Boolean Algebra, because neither the Contradiction Law nor the Middle Excluded Principle holds. Turning to the first mentioned definitions, both proofs can be expressed easily, by counter-examples, in an algebraic or geometric way.

Fuzzy Rules are linguistic IF-THEN constructions that have the general form "IF A THEN B", where A, B are propositions containing linguistic variables. A is called the premise, or antecedent, and B is the consequent (or action) of the fuzzy rule. In effect, the use of linguistic variables and fuzzy IF-THEN rules exploits the tolerance for imprecision and uncertainty.

In this respect, Fuzzy Logic imitates the ability of the human mind to summarize data and focus on decision-relevant information. For these reasons, it will be a very interesting way of advance, as an interesting access to many new fields of the Mathematics.

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