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SHORT COMMUNICATION

Determination of the Sign of the Spatial Curvature of the Universe Based on Friedmann's Equations and Experimental Data

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Abstract

It is a commonly accepted fact that the spatial curvature of the universe is very small or zero. Mission Planck measured a value very close to zero for the curvature term in the Friedmann equation. Another issue is determining its sign. From a theoretical physics perspective, the value of k can be -1, 0, +1, and depending on this value, the sign of the curvature term in the Friedmann equation changes. To date, no commonly accepted conclusion has been reached regarding this value. Likewise, the meaning of this sign implies different characteristics of the universe: finiteness, infinity; boundaries, unlimitedness, etc. In this work, we propose a value of k = +1, which is the same as a positive sign for the value of the spatial curvature, which makes the experimental results obtained, regarding the change in trend of the Hubble parameter 6 billion years ago, consistent with the Friedmann equations.

Keywords: Cosmos, General Relativity, The Universe on Large Scales, Spatial Curvature.

1. Introduction

Given the Friedmann equations, [1], of the FLRW metric, (ρ = Joules/m^3):

$H^2 = (a'/a)^2 = 8\pi G\rho/3c^2 + \Lambda c^2/3 - kc^2/a^2$

$(a''/ a) = - 4\pi G/3c^2 (\rho+3p) + \Lambda c^2/3$

2. Equation that Relates the Derivative of the Hubble Parameter with Pressure, Energy Density and the Curvature Spatial

According to the Friedmann equation, the equation of state, and taking into account the expression for the derivative of H, we have:

$H' = (a'/a)' = (a''/a) - (a'/a)^2$

$H' = (- 4\pi G/3c^2 (\rho+3p) + \Lambda c^2/3)- (8\pi G\rho/3c^2 + \Lambda c^2/3 - kc^2/a^2)$

$H' = - 12\pi G(\rho + p)/3c^2 + kc^2/a^2$

equation of state: $w = p/\rho$

$H' = - 12\pi G\rho(1+w)/3c^2 + kc^2/a^2$

3. Study of H During The Universe Epochs using this Equation

Let's study the sign of this equation in the different eras that make up the history of the universe. That is, during the radiation era, the matter era, and the dark energy era, which is the current one. Each of these eras is characterized by a different value of the equation of state parameter , w, (w=+1/3 in the period of universe dominated by radiation, w = 0 in the period of universe dominated by matter, w= -1 in the period of universe dominated by dark energy), and also according to experimental data [2], currently the term $\Omega_k = kc^2/ a^2$ is very small, and its sign is still undetermined.

Furthermore, according to recent experimental data, a trend change in the value of H has been observed as a function of the age of the universe, going from decreasing to increasing since 6 billion years ago. We, studying the equation derived from the values of w and these data, conclude that all these data are fully consistent with the FLRW metric as long as k = +1 is

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fulfilled, ruling out the other two possible values of k: 0 and -1. We explain this conclusion below:

First, we study the sign of H' according to the different values of w at different times in the universe. We summarize these calculations and reasoning in the following table:

Equation	
$H' = -12\pi G\rho(1+w)/3c^2 + kc^2/a^2$	
Study according to the values of w	
Period of the universe dominated by radiation, from its beginning until the year 360,000.	$w = +1/3$ $H' = -12\pi G\rho(1+1/3)/3c^2 + kc^2/a^2$
Period of the universe dominated by matter, from year 360,000 to 6 billion years ago. $w = 0$ $H' = -12\pi G\rho/3c^2 + kc^2/a^2$	
Period of the universe dominated by dark energy, from 6 billion years ago until its end $w = -1$ $H' = kc^2/a^2$	
Study according to the values of k	
Considering that the curvature term of the Friedmann equation is small compared to the other terms of the equation, the following will be fulfilled:	
	$k = -1$ H´ will be negative in all epochs, therefore H will always be decreasing.
	$k = 0$ H´ will be negative in the epochs dominated by radiation and matter and so H will be decreasing in the epochs dominated by radiation and matter and H´ will be zero in the epoch dominated by dark energy, so H will be constant in the epoch of the universe dominated by dark energy
	$k = +1$ H´ will be negative in the epochs dominated by radiation and matter and thus H will be decreasing in the epochs dominated by radiation and matter, H´ being positive in the epoch of the universe dominated by dark energy and therefore H will be increasing in the epoch dominated by dark energy.As observed experimentally

From all this we conclude that the only possible option according to the experimental data is that k = +1, that is, the spatial curvature is positive.

4. Conclusion

Using the Friedmann equations and the qualitative behavior of the Hubble parameter as a basis for its change from decrease to increase over the past 6 billion years, we have shown that of the three values that the curvature parameter k of the Friedmann equation can take (+1, 0, and -1), it necessarily takes the value k = +1. This implies a positive sign in the spatial curvature of the universe. To demonstrate this, we have obtained the equation for the derivative of the Hubble parameter as a function of the energy density, the equation of state parameter and the curvature term. This value of the equation of state, w, is determined for each period of the universe, and with it, we have been able to determine the sign of the derivative of the Hubble parameter for each period. It is evident that a transition from decrease to increase of the Hubble parameter only occurs if k = +1. Therefore, we believe that the spatial curvature has always been positive, which implies that the universe has the shape of a three-dimensional hollow sphere, is finite, has no boundaries, etc.

This study carried out here also clarifies the causes of the anomalous behavior of the Hubble parameter since 6 billion years ago, according to our equation of the derivative of the Hubble parameter, this behavior is a consequence of two causes, the first is dark energy and the second is the positive sign of the spatial curvature, without both this increase in the value of the Hubble parameter would not have occurred when passing from the era of the universe dominated by matter to the era of the universe dominated by dark energy, that is, 6 billion years ago.

5. References

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