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SHORT COMMUNICATION

Universe Expansion in Spatially Flat Universes

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Abstract

The expansion of the universe is a first-level issue in astrophysics. Knowing how it expands, its scale factor, allows us to solve a multitude of problems that we could not solve otherwise. Experimental data concerning the curvature of our universe show us that its spatial curvature is very close to zero, that is, our universe can be considered spatially flat. The most relevant cosmological models take this data into account. Here we have studied the expansion in a supposed universe model with FLRW metric and zero spatial curvature and we have come to the conclusion that in this assumption the expansion is always linear. Thus, we have shown that the scale factor in spatially flat universes with FLRW metric, is a linear function.

Keywords: General Relativity, Cosmos, Spatially Flat Universe.

1. The Scale Factor "a", in a Spatially Flat Universe

Given the Friedmann equations of the FLRW metric [1]:

$$H^2 = \left(\frac{a'}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2}$$

$$\left(\frac{a''}{a}\right) = -\frac{4\pi G}{3}\left(\rho + 3p/c^2\right)$$

We are going to show that for universes with FLRW metric and zero spatial curvature, k = 0, a´´ = 0 is fulfilled, that is, "a" is a linear function.

Let the FLRW metric be in coordinates (t, x^1, x^2, x^3) where "t" is the commoving time and x^i are the spatial coordinates, (c = 1).

$$ds^2 = dt^2 - a(t)^2(g_{\mu\nu} dX^\mu dX^\nu)$$

In a spatially flat universe, k= 0, the 3D hypersurface corresponding to each section of cosmic spacetime is the Euclidean space R^3.

We are looking for a coordinate transformation that will convert this metric into a conformal metric. We make the following coordinate change:

$dt = d\tau .\ a(\tau)$

$d\tau = a(t)^{-1} dt$

$a = a\ (t(\tau))$

$a(\tau) = dt/d\tau$

The conforming metric will be:

$ds^2 = a(\tau)^2(d\tau^2 - g_{\mu\nu}\ dx^\mu dx^\nu)$

Where the scale factor a(τ) = a(t(τ)) is now a function of conformal time. Conformal time is not the proper time of any particular observer, but these coordinates have some advantages, such as making it clear that FLRW metrics with k = 0 is a locally conformally flat metric. In a conformally flat metric, the curvature tensors are zero.

According to reference [1] in this metric the term $R_{\tau\tau}$ of the Ricci tensor is given by:

$R_{\tau\tau} = 3((a(\tau)''/a(\tau)) - (a(\tau)'^2/a(\tau)^2))$

We show below that: $a''(t) = 0$;

$a(\tau)' = da(\tau)/d\tau = (da(t(\tau))/dt).\ (dt/d\tau) = a(t)'.\ a(\tau).$

$a(t)' = a(\tau)'/a(\tau)$

$a(t)'' = d(a(t)')/dt = d(a(\tau)'/a(\tau))/dt = (d\ (a(\tau)'/a(\tau))/d\tau).\ (d\tau/dt) =$

$= ((a(\tau)''.\ a(\tau) - a(\tau)'^2)/a(\tau)^2).\ (1/a(\tau)) = R_{\tau\tau}/(3a(\tau)) = 0$

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Thus, we have shown that for spatially flat universes in the FLRW metric it is true that;

$a(t)'' = 0$

2. Conclusion

The FLRW metric that determines the evolution of the universe is based on the Cosmological Principle (the universe is homogeneous and isotropic at very large scales) and on Weyl's Postulate (the universe behaves like a perfect fluid whose components move as time geodesics without intersecting each other). This metric is specified in two equations, the Friedmann equations, in which the curvature term Ω_k plays an essential role in its resolution. This term has recently been measured [2] and it turns out to be very small, which leads to the belief that the spatial curvature of the universe is zero or very close to zero. That is why most current models that try to explain the universe and its evolution are based on this assumption. We have studied the FLRW metric in universes with zero spatial curvature and through a coordinate transformation we have converted it into a conformal metric. In this metric, a universe with zero spatial curvature presents a conformally flat metric. Taking this into account, we have shown that the expansion factor, "a(t)", in a universe with FLRW metric and zero spatial curvature, k=0, is a linear function. Our universe is a universe that behaves in large dimensions like a universe with FLRW metric and also its curvature is zero or very small, so it is expected to have a linear expansion or very close to it, however this is not the case, our universe has been accelerating for about 6000 million years and is not therefore in a linear expansion. It must be taken into account that in our real universe there is dark energy, we know very little about it and for this reason we have not introduced it adequately in the FLRW metric yet. Our result indicates the great importance of dark energy in the expansion of the universe to interpret some physical phenomena, including the expansion.

3. References

1. Bert Janssen. Theory of General Relativity. Granada University. Degree in Physics, 4º course. September 2013
2. Planck 2018 results. VI Cosmological parameters. Astronomy & Astrophysics manuscript. Vol 641. September 2020.