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# A Solution to the Dark Matter Problem

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**ABSTRACT**

*This is the first of two related papers on dark matter and dark energy. In this workalaw, $\mathbf{L_1}$, is proposed which tweaksNewton's Law of Gravity by the addition of an infinitesimal, intuitively reasonable term involving v/c. While the proposed perturbation barely affects the motion of planets and stars in the short run, the effect can be significant concerning these movements when measured over time spans running into the millions of years. In this case it is shown the application of $\mathbf{L_1}$ explains the dark matter problem.*

## INTRODUCTION

This work proposes a theoretical change in Newton's Law of Gravity (**NLG**) which obviates the need to invent dark matter forces to explain the motions of stars in galaxies. This law is covered in **Part 1.**Also, an important obvious corollary$\mathbf{C_1}$is offered as an immediate consequence which is especially apropos to the dark matter problem. $\mathbf{L_1/C_1}$is then applied in **Part 2** to orbiting bodies and to Mercury in particular. Following this the dark matter problem is resolved in **Part 3**.

## REFORMULATING NLG AS $L_1$

### Introduction

The theory in this work requires the reformulation of **NLG**as$\mathbf{L_1}$by making a very small, intuitive modification involving *v/c*. This law has virtually no effect on the motion of bodies in the short run, but a major effect over extremely long time spans. An important corollary $\mathbf{C_1}$ is added as an obvious application which is useful in resolving the dark matter problem.

### The Replacement of NLG with $L_1$

In this section **NLG** will be modified by law$\mathbf{L_1}$, and corollary $\mathbf{C_1}$ will be added as an immediate consequence. This theory examines the force exerted by a gravity "ray" sent at time *t* by a moving body of mass *M* and subsequently received by a moving body of mass *m*. In the analysis **IFR**(*t*) is defined as the inertial frame of reference of *M* at the ray emission time *t*. The term, "ray", is used here to indicate the gravitational field emitted over an infinitesimal period of time. The following assumptions are made:

### Assumptions

- All calculations are based on **IFR***(t)*.
- The ray travels from *M* to *m* at the velocity of light.
- The exerted force from *M* on *m* is in the opposite direction of the movement of the ray when it hits *m*.
- There is a reduction in this force when *m* is moving away from and v.v. when it is moving toward *M*.

When *M* and *m* are permanently stationary in a given inertial frame of reference, say **IFR***(t)* = $\mathbf{IFR}_0$, then the gravitational force, ***f***, is given by

$\boldsymbol{f}=\boldsymbol{f}_0$, where$\boldsymbol{f}_0$ obeys **NLG** for stationary bodies, as follows:

$$\boldsymbol{f}_0 = -GMm\mathbf{u} / r^2 \tag{1.2.1}$$

In this formula ***r*** is the constant vector running from *M* to *m* as measured in $\mathbf{IFR}_0$, and ***u*** is a unit vector given by $\boldsymbol{u} = \boldsymbol{r}/r$. The force is attractive in the direction of $-\boldsymbol{u}$. Now suppose in the more general situation that a ray is sent at time *t* from a moving body *M* to a moving body *m*, and the inertial frame of reference of *M* at the instant of the emission is **IFR***(t)*. Further suppose the ray travels at velocity *c* and hits *m* at a future position $\boldsymbol{r}(t+\Delta t)$at time=$t+\Delta t$, all as measured in **IFR***(t)*.It is assumed the exerted force, ***f***, at the instant of impact is in the direction of $-\boldsymbol{r}(t+\Delta t)$. If at this instant *m* is traveling at velocity $\boldsymbol{v}(t+\Delta t)$, which is at an angle $\varphi$ to $\boldsymbol{r}(t+\Delta t)$. The following general law of gravity, $\mathbf{L_1}$, is postulated:

Law $L_1$

$$\boldsymbol{f} = \boldsymbol{f}_0( 1- \alpha v \cos(\varphi) /c ) \tag{1.2.2}$$

In the above formulation $\alpha$ is a dimensionless constant which is assumed to be positive. This constant is discussed but not specifically evaluated in this work. Based on the analysis of the planet Mercury it will be clear that $\alpha \le 1$. From **(1.2.2)** it is convenient to define $V$ as follows:

$$V= v \cos(\varphi) \tag{1.2.3}$$

Then **(1.2.2)** can be restated as follows:

Law $L_1$

$$\boldsymbol{f} = \boldsymbol{f}_0( 1- \alpha V/c ) \tag{1.2.4}$$

It is noted that $V$ is the scalar component of the velocity of $m$ in the direction of the ray at the instant of impact, and that all quantities are evaluated in **IFR**$(t)$. In**(1.2.3)** $\varphi$ is the angle between $\boldsymbol{r}(t+\Delta t)$and $\boldsymbol{v}(t+\Delta t)$. If at the instant of impact $\boldsymbol{v}$ is precisely in the direction of $\boldsymbol{r}$, then $\varphi$=0 and $V$=$v$. Conversely, if $\boldsymbol{v}$ is in the-$\boldsymbol{r}$ direction, then $\varphi=\pi$ and $V$=-$v$. On the other hand, if $\boldsymbol{v}$ is orthogonal to the ray, then $V$=0 and the exerted force is $\boldsymbol{f_0}$.It is assumed that $V/c$ is small, so that $\boldsymbol{f}$ is a linear perturbation of $\boldsymbol{f_0}$.In the unusual event that $V/c$ happens to be large, it may be that **(1.2.4)** may need to be amended by adding on nonlinear terms.

It is seen that**$L_1$** differs very little from **NLG**. But the miniscule difference will prove to be useful in explaining the motions of planets and stars over very long time spans. It is important to note that **$L_1$** does not deal with situations involving the effect of gravity on photons, where $V/c$~1.It is also noted that the connection between **$L_1$**and the General Theory of Relativity (**GTR**), if any, is not specified. Both theories may be valid, and perhaps they will somehow be combined at some future date.

To better understand **$L_1$**think of the gravitational field emanating from $M$ and hitting $m$ later on. In general, the amount of field flowing past $m$ varies with the velocities of the two bodies. It is argued that the gravitational force stemming from the field that $M$ exerts on $m$ will be less if $m$ is moving with the field than if it is moving against it. This idea explains the negative term in **(1.2.4)**, where it is assumed that $\alpha>0$. These comments lead to the following important corollary **$C_1$**:

***Corollary $C_1$***

When $m$ is moving away from $M$ at the ray arrival (i.e., cos($\varphi$)>0), the gravitational attraction is reduced due to the $\alpha$ term and therefore $m$ is slowed less than under **NLG**. Alternatively, when $m$ is moving toward $M$, the gravitational attraction is increased and therefore $m$ is accelerated more than under **NLG**. Thus, in either case the speed of $m$ is increased relative to what it would be under **NLG**.

From**$C_1$** planets and stars moving in any direction, other than in a circle, under the primary influence of a single large mass $M$ will move faster everywhere than they would if only **NLG** were in effect. It is noted that the energy to increase the velocity of $m$ comes from the gravitational field of $M$ and not from any dark energy or dark matter forces. As will be explained in more detail later on, this **$L_1$/$C_1$** combination is the basis behind the solution to the dark matter problem.

Based on the following analysis, it is tempting to conjecture that $\alpha=1$. First, assume that the gravitational force on $m$ is $\boldsymbol{f}$, which differs from the Newtonian force $\boldsymbol{f_0}$ as a result of the amount of the gravitational field flowing by it, as follows:

$$\boldsymbol{f} = \boldsymbol{f_0} \text{ x (gravity flow past } m) / \text{(gravity flow when } V=0) \tag{1.2.5}$$

In time $dt$, the actual flow by $m$ is proportional to $(c\text{-}V)dt$, and the flow when $V=0$is proportional to $cdt$. Thus, **(1.2.5)** implies that:

$$\boldsymbol{f} /\boldsymbol{f_0} = (c\text{-}V)dt / cdt= 1 - V/c \tag{1.2.6}$$

From **(1.2.6)** and **(1.2.4)**it might be conjectured that $\alpha$=1. While this may in fact be true, the upcoming analysis concerning the motion of the planet Mercury makes it more likely that $\alpha$<<1.

## Similarity between Gravitational and EM Fields

In Aucamp [1] a theory of electromagnetism (**EM**) is developed which is essentially identical in formto**$L_1$**, at least in the linear perturbation case involving small $V/c$. Instead of $\boldsymbol{f_0}$ in **(1.2.1)**, a similar coulomb law equation for the force $\boldsymbol{F_0}$exerted between two stationary charges, $q_1$ and $q_2$ is given as follows:

$$\boldsymbol{F}_0 = q_1 q_2 \boldsymbol{u} / (4\pi\varepsilon_0 r^2) \tag{1.3.1}$$

Then the linear law**$L_1$**for the electric field force $\boldsymbol{F}$ is postulated as:

$$\boldsymbol{F}= \boldsymbol{F}_0( 1- \alpha v \cos(\varphi) /c ) \tag{1.3.2}$$

This force law is very similar to the proposed **$L_1$**gravitational force law. Defining $V=v \cos(\varphi)$ yields the same form as**(1.2.4)**:

$$\boldsymbol{F}= \boldsymbol{F}_0 ( 1- \alpha V /c ) \tag{1.3.3}$$

In Aucamp[1] the following conclusions are theoretically and experimentally shown: (a) magnetic forces are in reality electric field forces,(b) Maxwell's force laws and his equation for $c$ can be derived from **(1.3.3)**, and (c) $\alpha$=3/2. Though the value of $\alpha$ is mathematically and experimentally shown to be 3/2 for **EM** forces, this doesn't mean the same value applies to gravitational forces because these two fields are different.

## VIRTUALLY ELLIPTICAL ORBITS

### Introduction

$\mathbf{L_1/C_1}$ will be applied here to the problem of determining the velocity and orbit time of a planet of mass $m$ circling a star of mass $M$ in the case when the motion is virtually elliptical. The analysis applies equally well to the motion of a star orbiting a black hole center in a galaxy. The derived results will be informative in the study of the dark matter problem later on, where more details will be provided besides those found here. For convenience, $m$ will be termed a planet and $M$ a star.

It is assumed the planet is situated at the ellipse perihelion and is all set to execute an almost perfect elliptical orbit. It will now be shown how a positive value of $\alpha$ increases the velocity everywhere and therefore decreases the overall transit time. Consider a planet orbiting a star in an almost perfect ellipse and suppose $v=v_0$ is the current velocity at the perihelion. At this point $\boldsymbol{v}$ is orthogonal to the radius from $M$, where

cos($\varphi$)=0,$V=0$, and $v$is at a maximum. Now let $\alpha>0$ come into play, and let $\Delta v$ be the increase in the velocity over the upcoming orbit, where $\Delta v=0$if$\alpha=0$. It is noted that $\Delta v$ is very small because $\alpha V/c$ is assumed to be small. Suppose also that$T_0$is the orbit time if $\alpha=0$, and let $T_0+\Delta T$ be the time to complete the upcoming orbit if $\alpha>0$.As $\Delta v>0$ because of $\mathbf{C_1}$, then it will turn out that $\Delta T$<0.The derived results will then be applied to the movement of the planet Mercury.

Though the following assumptions concerning Kepler's laws are not strictly true because $\alpha>0$, it is argued they are sufficiently true to warrant their adoption because the orbit path is assumed to be virtually elliptical, where only a very small perturbation is due to the $\alpha$ term in $\mathbf{L_1}$. The effects of other perturbations are neglected here, such as those due to the forces exerted by other planets or to **GTR**, if any. As these effects are assumed to be small, they can presumably be added on later to get a total. The following assumptions will be made:

### Assumptions

- The orbit is almost precisely an ellipse.
- $M$ is essentially stationary, where $M$>>$m$.
- The effects of other bodies and **GTR** are neglected.
- $\mathbf{L_1/C_1}$ is valid.

Though it is assumed the movement of $m$is almost precisely along the path of a pure ellipse, it is also assumed an imperceptible force due to $\alpha>0$speeds up the body relative to that as determined by **NLG**. Though the movement is altered very slightly away from a pure ellipse, it is assumed here that the total velocity increase, $\Delta v$, over the orbit is essentially the same as it would be if the path were a pure ellipse. That is, it is argued that the assumption of $m$ moving on the ellipse rather than slightly off it will not significantly affect the calculation of $\Delta v$.

### Introduction to the Theory of Δv over One Orbit

Assume a planet of mass $m$ orbits a star of mass $M$, but the results apply equally to a star orbiting a black hole. The motion is depicted below in **Figure 1**, where the movement is counter clockwise around the star which is fixed at $\mathbf{F_1}$.The two foci are at $\mathbf{F}_1$, where the assumed stationary star is located, and at $\mathbf{F_2}$. Both foci are equidistant from **O**, and the distance from **O** to each is $\varepsilon a$, where $\varepsilon$ is the eccentricity. The semi-major axis with length=$a$ is shown as the horizontal line from the center **O** to $\mathbf{A}_2$ on the left and equally from **O** to $\mathbf{A}_1$ on the right. The perihelion is at $\mathbf{A_2}$, and the aphelion is at $\mathbf{A_1}$.The semi-minor axis of length $b$ runs vertically from **O** to $\mathbf{B}_1$ and equally from **O**to$\mathbf{B}_2$. An arbitrary point with coordinates $x$ and $y$ with respect to $\mathbf{F_1}$is shown at $\mathbf{C}_1$, which is at an angle $\theta$ to the major axis. The radii $r_1$ and $r_2$ are the distances from$\mathbf{F}_1$to $\mathbf{P_1}$and $\mathbf{F}_2$ to $\mathbf{P_1}$, respectively. The point$\mathbf{P}_2$ is also indicated as the symmetrical cousin of $\mathbf{P}_1$. The counter-clockwise distances around the ellipse to any given point, such as $\mathbf{P_1}$, is measured from $\mathbf{A_1}$.

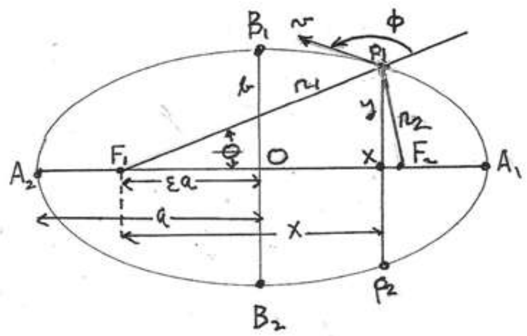

**Figure1**. *Purely Elliptical Orbit*

Certain properties of ellipses are well-known and will not be derived here. Three of them are as follows:

$$r_1 + r_2 = 2a \tag{2.2.1}$$

$$(x - \varepsilon a)^2/a^2 + y^2/b^2 = 1 \tag{2.2.2}$$

$$b = a(1-\varepsilon^2)^{1/2} \tag{2.2.3}$$

From **(2.2.3)** it is seen that the ellipse is a circle when $\varepsilon$=0.Since $v/c$ is assumed to be very small, $\mathbf{L_1}$as given by **(1.2.4)** will be used to solve the problem. As the star is assumed to be virtually stationary, this greatly simplifies the evaluation of the force $\boldsymbol{f}$ exerted by it on the planet because the gravitational field in this case is essentially non-varying. If $m$ were also motionless, which it is not, then the force, $\boldsymbol{f_0}(t)$,that $M$ would exert on $m$ would be given by**(1.2.1)** as follows:

$$\boldsymbol{f_0}(t) = - G\,Mm\boldsymbol{u}(t) / r^2(t) \tag{2.2.4}$$

In **(2.2.4)** $\boldsymbol{r}(t)$ is the vector from the star to the planet at time $t$, and $\boldsymbol{u}(t)=\boldsymbol{r}(t)/r(t)$. However, as $m$ is in fact moving, then from **(2.2.4)** and $\mathbf{L_1}$the force that the star exerts on the planet is given as:

$$\boldsymbol{f}(t)=- G\,Mm\boldsymbol{u}(t)\,[1-\alpha V/c] / r^2(t) \tag{2.2.5}$$

It is noted that $V=v\,cos(\varphi)$, where $v$ is the counter-clockwise scalar velocity of the planet with respect to the assumed stationary star and $\varphi$ is the angle between $\boldsymbol{r}$ and $\boldsymbol{v}$, *as* shown in **Figure 1**. It is also noted that dropping the term involving $\alpha V/c$ in **(2.2.5)** results in a purely Newtonian force that yields a purely elliptical path. As $V/c$is assumed to be very small, and as it will be argued later $\alpha \le 1$, then $\boldsymbol{f}(t)$will differ from $\boldsymbol{f_0}(t)$by only a miniscule amount. This miniscule difference, call it $\Delta\boldsymbol{f}(t)$, is what is of importance in this study. It is defined as follows:

$$\Delta\boldsymbol{f}(t)=\boldsymbol{f}(t)-\boldsymbol{f_0}(t)= G\,Mm\boldsymbol{u}(t)\,[\,\alpha V/c\,] / r^2(t) \tag{2.2.6}$$

As $V=v\cos(\varphi)$,plugging this into **(2.2.6)** and noting that $\boldsymbol{u}\,(t)$ depends only on $\boldsymbol{r}$ yields:

$$\Delta\boldsymbol{f} = G\,Mm\boldsymbol{u}(\boldsymbol{r})\alpha v\cos(\varphi)/(c\,r^2) \tag{2.2.7}$$

Since the component of $\Delta\boldsymbol{f}$ in the direction of increasing counterclockwise $s$ along the ellipse is being sought, and since the scalar value of $\boldsymbol{u}(\boldsymbol{r})$in this direction is $\cos(\varphi)$, then the scalar increase, call it $\Delta F$, in the force over and above the Newtonian force is found from **(2.2.7)** as follows:

$$\Delta F=G\,Mm\alpha v\cos^2(\varphi)/(c\,r^2) \tag{2.2.8}$$

It is concluded from **(2.2.8)** that $\Delta F$>0 everywhere except where $\cos(\varphi)$=0. Thus, $m$ speeds up at every point where $cos(\varphi)\neq 0$ relative to the pure Newtonian motion, and this conclusion agrees with corollary $\mathbf{C_1}$.Note also that the $\Delta F$*'s* at two related points such as $\mathbf{P_1}$ and $\mathbf{P_2}$ in **Figure 1**are equal, and it is therefore clear that the increase in velocity over and above **NLG** will be the same over segments $\mathbf{A_1A_2}$and$\mathbf{A_2A_1}$.

## Evaluation of Δv from ΔF

In this section the velocity increase, $\Delta v$, in one virtually elliptical orbit as measured from perihelion to perihelion will be analyzed from $\Delta F$ as given by **(2.2.8)**. It is reiterated that when $\alpha=0$ the orbit is a standard ellipse and that $\Delta v=0$. Accordingly, $\Delta v$ is totally due to $\Delta F$. It is therefore convenient in the analysis to neglect the classical Newtonian force and just consider the $\Delta F$ perturbation. Then from $dv=fdt/m$:

$$\Delta v = \oint \Delta F\,dt / m \tag{2.3.1}$$

Inserting $\Delta F$ as determined by **(2.2.8)** into **(2.3.1)** yields, after a little algebra, the following:

$$\Delta v = [\alpha G\,M / c] \oint \cos^2(\varphi) v dt / r^2 \tag{2.3.2}$$

The closed integral in **(2.3.2)** is carried out over one orbit. As $\boldsymbol{v}$ is in the direction of the ellipse, then $vdt=ds$, and **(2.3.2)** becomes:

$$\Delta v =[\alpha G\,M / c]\oint \cos^2(\varphi)\,ds/\,r^2 \tag{2.3.3}$$

It is convenient to normalize all variables involving length by dividing them by the semi-major axis length, $a$. When this is done, **(2.3.3)** becomes:

$$\Delta v= [\alpha G\,M/(ac)]\oint_{-L}^{L} \cos^2(\varphi)\,dL/ R^2 \tag{2.3.4}$$

In this formulation the closed path integral is that of a "standard ellipse" with a semi-major axis of $a$=1. The total path length is 2$L$, which unfortunately is not analytically available as a function of $\varepsilon$(other than via an infinite series or by numerical integration). The dimensionless variable, $R$, is defined as $R=r/a$. As the integral in **(2.3.4)** is only a function of the eccentricity $\varepsilon$, let $I(\varepsilon)$ be its value. Then:

$$\Delta v=[\,I(\varepsilon)\alpha G\,M]/[ac\,] \tag{2.3.5}$$

where

$$I(\varepsilon) = \oint_{L}^{L} \cos^2(\varphi)\,dL/ R^2 \tag{2.3.6}$$

The value of $I(\varepsilon)$can readily be found via a computer program by numerical integration for any value of $\varepsilon$. For Mercury, $\varepsilon$=.20563069, and $I(\varepsilon)$ is:

$$I(\varepsilon)=I(.20563069)=.1379675126(\text{Mercury}) \tag{2.3.7}$$

It is noted from **(2.3.5)** that the increase in velocity will result in a precession, which is over and above that from other causes, such as from other planets or from **GTR**. All these causes, including the use of measuring devices on the Earth, make it difficult to experimentally determine the precession due to $\Delta v$ alone.

### Determination of ΔT

In this section the change $\Delta T$ in orbit time $T$ as measured from the initial perihelion to next perihelion will be determined as a function of $\alpha$. The reason $\Delta T$ is of interest here is because it contributes to the precession of the planet. As $\Delta v>0$, then $\Delta T<0$. It is known from Kepler's laws that:

$$a = [GMT^2/(4\pi^2)]^{1/3} \qquad \textbf{(2.4.1)}$$

It is also known that the initial velocity $v_0$ at the perihelion is given by:

$$v_0=(2\pi a/T)\,[(1+\varepsilon)/(1-\varepsilon)]^{1/2} \qquad \textbf{(2.4.2)}$$

From **(2.4.1)** the following obtains for small $\Delta T$:

$$\Delta T = -\,T^2\Delta v\,/\,\{2\pi a\,[(1+\varepsilon)/(1-\varepsilon)]^{1/2}\} \qquad \textbf{(2.4.3)}$$

As all the quantities in **(2.4.3)** are known, $\Delta T$ can be determined. From **(2.3.5)** and **(2.4.3)**, after cancelling terms and rearranging, the result is:

$$\Delta T\,/\,T = -\Delta v\,/\,v_0 \qquad \textbf{(2.4.4)}$$

Thus, under the assumption that the movement is virtually along the ellipse, the infinitesimal fractional decrease in orbital time equals the infinitesimal fractional increase in orbital velocity, all as measured at the perihelion.

### Application to Mercury

In this section $\Delta v$ as given by **(2.3.5)** and $\Delta T$ by **(2.4.4)** will be found as a linear function of $\alpha$ for the planet Mercury, which has a relatively high eccentricity as compared to the other planets in the solar system. The constants that are used (in MKS) are:

$G$ = 6.6720 x $10^{-11}$

$M$=1.98910 x $10^{30}$

$c$=2.99792x$10^{8}$

$\varepsilon$ =.20563069

$T$ =7.600435x$10^{6}$sec (87.968 Julian days)

$a$ = 5.79086 x $10^{10}$

$v_0$ = 5.897672 x $10^{4}$

From a computer program:

$I(\varepsilon)$= .1379677126

$S$=3.599733x$10^{11}$ meters

The orbit time of 87.968 Julian days is viewed here as a fixed constant in this study, and $v_0$ and $a$ are determined accordingly from Kepler's laws. On inserting the above values into **(2.3.5)**, the following obtains:

$$\Delta v = 1.05469165\,\alpha\text{(meters/sec)} \qquad \textbf{(2.5.1)}$$

$$\Delta v\,/\,v_0 = 1.78831863\text{x}10^{-5}\alpha \qquad \textbf{(2.5.2)}$$

Then, from **(2.4.4)**:

$$\Delta T/T = -\,\Delta v\,/\,v_0 = -\,1.78831863\text{ x}10^{-5}\alpha \qquad \textbf{(2.5.3)}$$

Accordingly, based on the known assumed value of $T$ for Mercury:

$$\Delta T = -\,1.78831863\text{ x}10^{-5}\,T\alpha = -135.919998\,\alpha \qquad \textbf{(2.5.4)}$$

From these results, the precession of Mercury per orbit due to the orbital time decrease from **L₁**isgiven as:

$$\text{Precession (radians/orbit)} = v_0|\Delta T|/\,r_0 = 1.742\,60209\text{ x }10^{-4}\alpha \qquad \textbf{(2.5.5)}$$

It is noted here that $\Delta T<0$, so that the precession is positive, and $|\Delta T|$ must be used above in **(2.5.5)**, where $r_0=a(1-e)$. As 1 radian=206, 264.806 arcsec, then the precession in arcsec is:

$$\text{Precession (arcsec/orbit)} = 35.9437472\,\alpha \qquad \textbf{(2.5.6)}$$

Over one century, there are $N$=415.52078 orbits. Then the approximate precession per century due to the **L₁** speedup is very close to the multiple of $N$ times the precession/orbit, so the precession in arcsec/century is given as (415.52078)x(-35.9437472)$\alpha$=14924.1297$\alpha$. It is interesting that astronomers have calculated the actual precession of Mercury due to all causes with respect to an ICRF to be about 574 arcsec. If this result is accurate, then it is surmised that $\alpha$<<1.This is in contradiction to the possibility that $\alpha$ might be unity. As there are many twists and turns in the above analysis and computer program, and as the astronomical evaluation is based on **NLG**, it is uncertain at this point whether to assert anything definite about $\alpha$, especially concerning the $\alpha$=1.

### Application to Gravity Waves

From **L₁** gravity fields travel at $c$. Consider, for example, what happens when one star collides with another star. Instantaneously, the resulting two gravity fields undergo huge transitions which travel out into space at the speed of light. Based on the weakness of these fields millions of light years away, it is impressive that scientists have found a way to measure their presence. It is noted that these experiments assume that gravity fields travel at $c$, which is likewise assumed in**L₁**.

## DARK MATTER

### Introduction

The strange movement of stars in galaxies has been a perplexing and unresolved mystery for quite a while now, and the term, "dark matter", has been coined to indicate this lack of understanding. It is the view of many scientists that there must be some form of matter out in space that is responsible for these movements, though none has ever been detected. This problem primarily arosefromthe1975workof Vera Rubin on rotation curves in the Andromeda galaxy and the subsequent publication in 1976 by the team of Rubin, Roberts, Graham, Ford, and Thonnard [2]. One of the conclusions in that study was that stars in galaxies inexplicably often move in spiral patterns around a black holec.g., that their velocities tend to increase with the distance *D* from it, and that they then tend to level off. This behavior is in direct contradiction to **NLG**, where orbital velocities decrease with greater *D*, as is the case with planets in our own solar system. Subsequent studies by others have found similar results. The theory given here is based solely on $\mathbf{L_1/C_1/O}$, where the attached **O** refers to the orbital material in **Part 2**. The analysis will be directed toward providing reasons for these observations without the need for inventing dark matter forces.

### Intuitive Solution

Based on $\mathbf{L_1/C_1/O}$ the answer to the dark matter problem can be seen intuitively, as discussed below. The remaining sections will then be devoted to adding back-up theory. First, from $\mathbf{L_1/C_1/O}$ the velocity of a given star moving around a black hole of mass *M* will keep increasing at all points in its motion relative to what it would be if the speed were solely based on **NLG**. This is the case both when the star is slowing down as it moves away from *M* and when it is speeding up as it moves toward it. Thus, if the star happens to be moving in some kind of a complete orbit, there is a net velocity gain over it. This gain is added to all the speeds in the next orbit. Therefore, the orbital speeds increase along any radius line extending outward from the black hole center. This is one of the key observations of Rubin.

Second, the added force provided by $\mathbf{L_1/C_1/O}$ indicates the moving star will be pushed outside the locally elliptical orbit at all points where the velocity is not orthogonal to the gravitational ray. However, as the additional force decreases with the distance *D* from *M*, then the $\Delta v$ orbital gains become less and less. Thus, on the one hand there is an increase in velocity and a movement away from a pure ellipse which results in a spiral path which eventually escapes into outer space. On the other hand, since the velocities are leveling off, this indicates a movement that becomes more and more circular. Which of these two eventualities, whether a circle or a spiral, wins out in the long run is deemed to be a function of the current state, ($\boldsymbol{r}$, $\boldsymbol{v}$, and *M*), which is not ascertained in this work. To make matters even more complicated, *M* may increase in time as more and more material is added to it. If the spiral wins out, then according to classical physics the final escape path initially starts out as a parabola. Subsequently, from $\mathbf{L_1/C_1}$ the small increase in speed over and above **NLG** results in a slightly hyperbolic path.

The details covered in the remainder of this study are offered primarily as back-up filler material to the above intuitive remarks. While elliptical orbits are well-understood, the details in the case when the orbits are decidedly non-elliptical are more complicated, and the analysis in this latter case will therefore be mainly qualitative. The many problems which preclude more specific mathematical equations in the decidedly non-elliptical case include: (a) the value of $\alpha$ is currently unknown, (b) it is difficult to analyze paths which are spiral and not elliptical, (c) the time spans of interest run into millions of years, which make both theoretical and computer simulation studies difficult to undertake, (d) the set of initial conditions for the stars in any given galaxy is unknown, (e) *M* may increase significantly, and (f) the effects of the millions of other stars in a galaxy are difficult to handle mathematically.

The analysis in the remaining sections is divided into the following three phases of galaxy life spans:

- **Phase1** – initial movements which are virtually elliptical
- **Phase2** – non-elliptical but non-terminal movements
- **Phase3** – terminal movements which are often circular or hyperbolic

### Phase.1 Virtually Elliptical Movements

The key idea in **Phase 1**lies in the application of$\mathbf{L_1/C_1/O}$, where it is clear that a star of mass *m* gets a small gravitational push over and above the **NLG** force from the black hole center of mass *M*, no matter if *m* is moving away from *M* or toward it. In all three phases this push is the source of the energy that has perplexed scientists, and it occurs no matter if the movement is elliptical or spiral or anything else (other than a circle).It is noted that it is not assumed here that all stars at some point in time get to be in a **Phase 1**state. It may be that some stars enter the gravitational field of the given galaxy in a **Phase 2** state.

From **Part 2** the speed of $m$ increases at all points in a **Phase 1** orbit with respect to the speed determined by **NLG**, except at those points where $cos(\varphi)=0$. Thus, it is clear that the orbit will not be precisely elliptical. Due to the velocity increase, the **Phase 1**motion will be in a direction which is slightly outside the ellipse and away from $M$. Therefore, at the end of each orbitat the perihelion the actual motion will be at a slight angle to the semi-major axis, with the result that the overall path will be a slight spiral. As the speed relative to that determined by **NLG** keeps increasing over the length of each orbit, the path will become more and like a spiral. Also, the speed around each subsequent orbit will get more and more uniform because of the greater distances from $M$ and the higher starting velocities. As a result, on the one hand the path should become more and more circular, while on the other hand the path should become more and more like a spiral. So there would seem to be a tug of war going on between the spiral and circular movements.

At any rate, it is argued there are two possibilities for what will eventually happen when stars are in a **Phase 1** state: either the path will become circular or it will become a spiral which then ultimately escapes into outer space. It is therefore conjectured that there exists two mutually exclusive and collectively exhaustive regions, $\Psi_C(\mathbf{r},\mathbf{v},M)$ and $\Psi_S(\mathbf{r},\mathbf{v},M)$, in which a given star falling inside $\Psi_C$ at any time ends up in a circular path and inside $\Psi_S$ a spiral path. The mathematical analysis for the solution to this complex problem is not yet known to this author.

In **Phase 1** it is noted that the eccentricity decreases with each orbit because the velocity becomes more and more uniform. Thus, as orbits become more and more circular, $\Delta v$ per orbit becomes less and less, and it may very well be that all stars that happen at some time to be in a **Phase 1**state stay in that state forever and eventually end up moving in a circular path. In this case it may be that only those high velocity stars that have entered the galaxy in the $\Psi_S(\mathbf{r},\mathbf{v},M)$region eventually exit into outer space, where they initially move in somewhat parabolic paths which then become slightly hyperbolic due to $\mathbf{L_1/C_1}$.

### *Phase 1 Conclusions in Greater Detail*

In the case of planets circling around a stationary star it was shown in **Part 2**that the orbits are virtually, but not quite, elliptical. Extending this result to orbiting stars is **Conclusion #1**listed at the end of this section. Also derived in **Part 2**as given by **(2.3.5)** is that $\Delta v=I(\varepsilon)GM/(ac)$,where $I(\varepsilon)$ is a known monotonically increasing function of the eccentricity $\varepsilon$. This function can be calculated by a computer program. As the same result is assumed to apply to stars in galaxies in **Phase 1**, this is listed as **Conclusion #2**.

From the results which are similar to the Mercury study assume a given **Phase 1**star orbits around $M$ along a virtually elliptical path. As the star speeds up by a miniscule amount over each orbit, the actual path is not precisely elliptical. Letting $\Delta v_n>0$ be the total increase in velocity over the $n^{th}$ orbit, then this increase is added to the start of the next orbit, which means that the velocity during orbit $n+1$ exceeds the velocity during orbit $n$ at all corresponding points. This is **Conclusion #3**.

From **Figure 2** below a graphical analysis is offered which shows the early, virtually elliptical, orbit in **Phase 1**.This orbit is actually a spiral (the difference is greatly amplified in the drawing). Assume the black hole focus is at **F**, and the major axis runs from **A** to **C** along the $x$ axis. Also assume the star begins the $n^{th}$ orbit at a point **A'** which is almost precisely at **A** (again, greatly amplified in the drawing).The overall path is very close to that of an ellipse as given by the path **ABCDA**, but it actually starts at **A'** and ends at **A''**.As the orbit is not a perfect ellipse, the actual path begins along the dashed line starting at **A'** and ends up along the dashed line at **A''**, where these two points are almost precisely at **A**. It is noted the entire path falls outside the ellipse because the velocity increases more than it does under **NLG**. Thus, **A''** falls to the left of **A'**, and the slope at **A''** is consequently less steep than at **A'**. This is Conclusion **#4**.

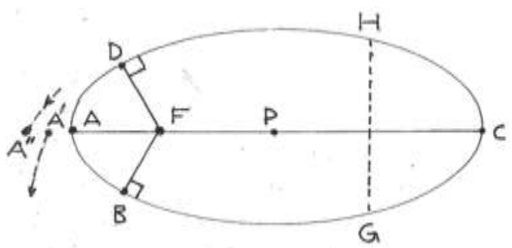


**Figure2**. *Phase1 (Early Orbit: Amplified at A)*

Now from **Figure 2** let $\Delta x_L = x(\mathbf{A''}) - x(\mathbf{A'}) < 0$be the algebraic distance from **A'** to the left at **A''**. Similarly, at the right extreme at **C** let

$\Delta x_R = x(\mathbf{C''}) - x(\mathbf{C'}) > 0$be the corresponding distance from **C'** to the right at **C''**, where **C'** and **C''** are defined in a manner similar to **A'** and **A''**. Then the total increase $\Delta a$ in the semi-major axis is$2\Delta a = \Delta x_R - \Delta x_L$, which implies the net increase in the semi-major axis over the orbit is given as follows:

$$\Delta a = (\Delta x_R - \Delta x_L) / 2 \qquad \textbf{(3.3.1)}$$

Next, it is conjectured from the overall symmetry that:

$$|\Delta x_L| = \Delta x_R \qquad \textbf{(3.3.2)}$$

Then, from **(3.3.1)** and **(3.3.2)**:

$$\Delta a = |\Delta x_R| = - \Delta x_R \qquad \textbf{(3.3.3)}$$

From **(3.3.3)** the center point of the perturbed path remains at **P**. Thus, the perturbed value $\varepsilon^*$ of $\varepsilon$ is found as the ratio of the distance from **F** to **P**, which is$a\varepsilon$, divided by the distance from **F** to **A''**, which is $a+\Delta a$, as follows:

$$\varepsilon^* = a\varepsilon / ( a+\Delta a) = \varepsilon / (1+\Delta a / a ) \qquad \textbf{(3.3.4)}$$

For small $\Delta a$, **(3.3.4)** can be closely approximated as follows:

$$\varepsilon^* = \varepsilon(1 - \Delta a / a ) = \varepsilon - \varepsilon \Delta a / a = \varepsilon + \Delta\varepsilon \qquad \textbf{(3.3.5)}$$

From **(3.3.3)** and **(3.3.5)**it is concluded that $\Delta\varepsilon = - \varepsilon \Delta a/a$. Thus:

$$\Delta\varepsilon/\varepsilon = - \Delta a/a < 0 \qquad \textbf{(3.3.6)}$$

As $\Delta\varepsilon<0$ in **(3.3.6)**this puts a limit on the amount of velocity increase a star can eventually attain when in a **Phase 1** state. While it is assumed the starting orbit is virtually an ellipse in this analysis, the orbit may eventually develop into something quite different. This is **Conclusion #5**.

In a manner similar to the orbit of planets analyzed in **Part 2**, it will be assumed that in the $n^{th}$ orbit of a star in **Phase1**the total velocity increase, $\Delta v_n$, will be positive. This will result in a starting velocity in the next orbit being increased by $\Delta v_n$. Thus, the velocity along any given orbit will be more evenly distributed than in the prior orbit. It is also noted this result also applies to **Phase 2** and **Phase 3** paths, and the explanation of this virtual constancy is the solution to one of the perplexing and unexplained discoveries by Rubin. This is **Conclusion #6**. For example, consider the planet Mercury. It is known the speed along a given orbit varies considerably around its path. But if a cumulative amount were added to all points in that orbit, the speed along its path would not vary as much.

### *Summary of Phase1 Conclusions*

- Orbits in **Phase 1**are virtually elliptical, but are actually slightly spiral.
- With respect to the velocity determined by **NLG**, $v$ increases at all points in each orbit (except those with $cos(\varphi)=0$).
- At each point in the $(n+1)^{st}$ orbit the speed exceeds the speed at corresponding point in the $n^{th}$ orbit.
- The ending slope of the orbit in **Figure 2** is at a slant to the beginning slope.
- **Phase 1** orbiting stars in galaxies (and planets in orbit about a star) move in slightly spiral planar patterns. These patterns generally start out being almost elliptical, but may develop into something decidedly different.
- The velocity distribution along the $(n+1)^{st}$ orbit is more constant than along the $n^{th}$ orbit, so in this sense the orbits in **Phase1** become more and more circular.

## Phase.2 Movements (Non-Elliptical but Non-Terminal)

In **Phase2** the spirals previously covered in **Phase1** have either grown so large that they are no longer virtual ellipses, or these stars have entered the galaxy in **Phase 2**.It is reiterated that it may turn out that this phase may not in fact be reachable from **Phase 1**. Instead, the orbiting star in **Phase 1** may continue to move in a more circular path and simply end up orbiting in a circle. In any case, the problems with **Phase 2**include: (a) the starting conditions vary, (b) the time spans in this phase may cover millions or billions of years, thereby making explicit formulas difficult to derive and computer simulations difficult to run, and (c) the spirals tend to get bigger and bigger and therefore become less and less like ellipses and more and more analytically complex. For these reasons the upcoming analysis is based on arguments which are more intuitive and qualitative, and not mathematical. It is hoped that future research will strengthen the theory in a more rigorous manner.

In **Phase2** the spirals previously analyzed in **Phase1** have grown so large or have begun so large that they are not virtual ellipses. An example situation of one orbit is shown below in **Figure 3**.

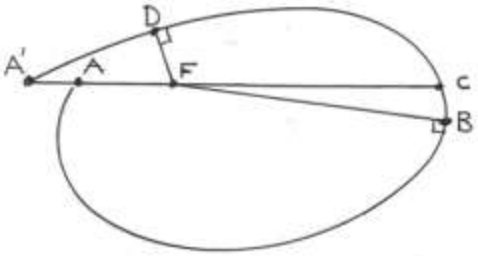


**Figure3.** *Example of a Phase 2 Orbit*

In this figure a star moves from **A** around the orbit, **ABCDA'**, where the black hole focus is at **F**. Corollary $\mathbf{C_1}$ ensures that the velocity increases around the orbit as compared to what it would be if only **NLG** were in effect. On the segments **AB** and **DA'** the distance from any given point to **F** is increasing, and therefore the attractive force is reduced from what it normally would be under **NLG**. The opposite is true along **BD**. Thus, when the star is moving away from **F**, the gravitational attractive force is less than as dictated by **NLG**, and v.v. when it is moving toward **F**. The net result is the star will speed up more than it would under **NLG** and therefore the initial slope exceeds the final slope, as shown in the figure. All of this is in agreement with corollary $\mathbf{C_1}$.Accordingly, it is concluded that stars in **Phase 2** orbits speed up with respect to the prior orbit, but at a lesser and lesser pace. The question therefore is whether or not the successive spirals will grow forever, or will they mature into a circle? As each orbit starts out with an overall $\Delta v$ increase with respect to the prior orbit, in this sense they will become more and more constant in velocity. The end result is therefore as follows:

### *Proposition Concerning the Phase 2 End Result*

- Either the ending path will be permanently circular
- Or the escape velocity $v_e$ will be reached and the path will initially become parabolic and subsequently slightly hyperbolic as the star moves into outer space.

It is noted that Rubin's rotation curves for **Phase 2** spirals show initial velocity increases and then the velocities tend to become somewhat constant. These features are in agreement with the above analysis. Another interesting aspect of galaxy formations is that stars sometimes exhibit a barbell pattern with two large conglomerations at either end of a long "stick". It is theorized that these groupings occur because there are a large number of stars attracting one another. Over time, a bunch of stars will tend to group and all rotate somewhat together. If two major groupings occur having essentially similar rotation speeds, it is conjectured they should be as far away from each other as is possible where the force of attraction is least. Thus, if there are two spiral groupings with essentially the same rotation speeds, they should be at either end of the stick.

## Phase.3 Terminal Movements

The **Phase 3** situation is exemplified in **Figure 4** below

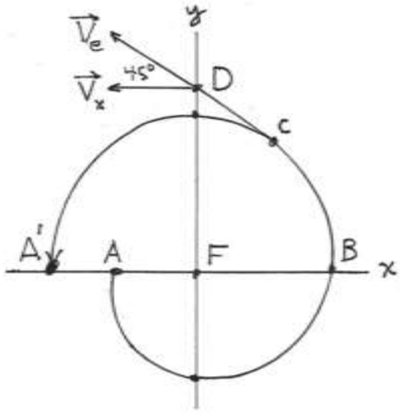


**Figure3.** *Phase 3 Example*

In this figure the movement of a star about a black hole at **F** is shown moderately late in its history. Consider the situation at point **C**, where the star is either in **Phase 1** and about to enter a terminal circular orbit or it is in **Phase 2** and close to exiting into outer space. In the latter case it moves to **D** and escapes. In the former case it moves to **A'** and thereafter moves in a circle. In either case the star moves faster and faster than it would under **NLG**, but the velocity increase may be very small because of the large distances and the somewhat circular movement. In the case of a circular ending, no further increase in energy is attainable. In the situation where the star escapes into outer space, the initial movement is parabolic. Then, due to the large distances, little energy from corollary $\mathbf{C_1}$is added thereafter, and the resulting movement becomes only slightly hyperbolic. Thus, the velocity remains somewhat constant in this phase, as noted by Rubin. As no formula is offered in this work concerning the equation for the movements of stars in **Phase2**, no analysis will be given here as to which of the two end states will be reached for any particular earlier state. However, as the velocity increases in each orbit, it is clear that the velocity around any given path will become more and more constant. Thus, there is less and less potential for a big $\Delta v$ over any given orbit, as observed by Rubin. So it comes down to whether or not the exit velocity is reached before the constant velocity situation.

If there is an eventual escape, assume it occurs at some point, say at **D** in **Figure 4**. Further assume the scalar radius from **F** to **D** is $r_e$. The escape velocity $V_e$ at **D** is shown, as is the velocity component $V_x$ orthogonal to $r_e$. It is well-known and easily proved that $V_e$ is the speed that yields a total energy $E$=0, where $E=U+K$, $U=-GMm/r$, and $K=mV^2/2$. The required exit velocity is therefore given as follows:

$$V_e = (2GM/r_e)^{1/2} \qquad \textbf{(3.5.1)}$$

It is emphasized that the star in question is moving in a planar spiral path. If the star were moving in a perfect ellipse at **D**, and if this point were at the perihelion where the elliptical velocity is maximized, then velocity $V_0$ would be given as follows:

Then velocity $V_0$ would be given as follows, where $V_0 = V_x$

$$V_0 = (GM/R_e)^{1/2} \qquad \textbf{(3.5.2)}$$

From **(3.5.1)** and **(3.5.2)** it is seen that:

$$V_e / V_0 = \sqrt{2} \qquad \textbf{(3.5.3)}$$

From **(3.5.3)** it is noted that the angle between the velocity vectors $V_e$ and $V_0$ should be 45 degrees, or about 45 degrees, as is shown in the figure. Thus, the following orbital result is conjectured:

### Spiral Escape Orbit Conjecture

It is conjectured that the escape path in the final spiral orbit will often lie at or near to a 45° angle to $r_e$ and there after follows a path which is slightly hyperbolic but essentially parabolic.

## Final Conclusions

In this work law $L_1$ is proposed which represents a small perturbation in **NLG**. Also, from $L_1$ an important corollary $C_1$ is set forth as an immediate consequence. Then the dark matter problem is resolved from $L_1/C_1$, as well as the results of the analysis of virtually elliptical orbits in **Part 2**. It is argued the invention of dark matter forces are not needed to explain the strange movements of stars in galaxies.

$L_1$ asserts the gravitational force given by **NLG** should be tweaked in a very small way which barely affects the motion of objects in the short run, but the perturbation can be important concerning the motions of planets in solar systems and stars in galaxies over very long time spans. From $C_1$ it is seen that a body of mass $m$ moving in a non-circular path under the gravitational influence of a large, essentially stationary mass $M$ will receive a small push from the gravitational field of $M$, over and above that given by **NLG**. In particular, $m$ is thereby slowed less when it is moving away from $M$ and speeded up more when it is moving toward it. In either situation there is a relative velocity increase over and above what would be the case with **NLG** alone. As a result there is an energy gain from the gravitational field of $M$ that has nothing to do with dark matter or dark energy forces. This energy gain is used to explain the dark matter force problem.

In conclusion, based on $L_1/C_1$, arguments are made which resolve the dark matter problem as due to natural causes.

## References


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