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A Single Equation Solution to the Electromagnetic Force Problem

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ABSTRACT

This is the first of a series of four related papers on electromagnetic forces, radiation, special relativity, and dark energy. A single equation comprising the entirety of electromagnetic force theory is intuitively developed in this work, and then analytically proved and experimentally verified. This solution to what is arguably the fundamental problem in EM theory determines the force that a moving charge exerts on another rmoving charge. The derived force is independent of the user's inertial frame of reference, and it explains well-known magnetic and emf experiments involving currents. Though Maxwell's equations involving forces and his formula for c are treated as example problems, they are still viewed as being very important.

PRELIMINARIES

Introduction

This work is the first of four related papers dealing with EM force theory, radiation, SR, and dark energy. In Part 1 the groundwork is laid for the remainder of the paper. The problem studied, which is viewed as the fundamental problem in EM theory, is to find the force that a charge,q1, moving at v1 exerts on a charge, q2, moving at v2, all as measured in an arbitrary fixed inertial frame of reference, IFR0. The resulting equation for the force, which is given by Equation (2.4.5),represents the entirety of EM force theory. This equation is intuitively developed in Part 2and then mathematically verified in Part 3. Experimental validations concerning the magnetic forces and emf's resulting from currents are provided in Parts 4 and 5, respectively, as well as the derivation of Maxwell's formula for the velocity of light. In the equations given below, u=r/r.

Several Classical Equations of Use in This Work

(1.2.1)$\boldsymbol{f}=q_1\, q_2\boldsymbol{u}\ /\ (4\pi\varepsilon_0 r^2)$ (Coulomb's force law)

(1.2.2)$\boldsymbol{E} = q\ \boldsymbol{u}/\ (4\pi\varepsilon_0 r^2)$(Coulomb's electric field law)

(1.2.3)$\boldsymbol{f}=q\boldsymbol{v}\mathrm{x}\boldsymbol{B}$(magnetic force law)

(1.2.4)$\boldsymbol{f} = q\ [\ \boldsymbol{E} + \boldsymbol{v}\mathrm{x}\boldsymbol{B}]$(Lorentz'slaw)

(1.2.5)$\boldsymbol{B} = [\mu_0/(4\pi)]\ q\ \boldsymbol{v}\mathrm{x}\ \boldsymbol{u}\ /r^2$(Biot-Savart law)

(1.2.6)$c = \sqrt{\ } 1/(\varepsilon_0\mu_0)$ (Maxwell's equation for c)

(1.2.7)$\oint \boldsymbol{E}\cdot d\boldsymbol{L} = -d(\iint \boldsymbol{E}\cdot d\boldsymbol{S})/dt$(Faraday's law)

The Implied Classical Solution to the Force Problem

Consider Lorentz's law as given by **(1.2.4)**, which is often viewed as the solution to the problem concerning the total **EM** force on a charge. If there is only a single charge, q_1,which exerts a force on a single charge,q_2, then $\boldsymbol{E}$ in**(1.2.4)** can be re-written by **(1.2.2)** and **B**by **(1.2.5)**. The resulting implied force, $\boldsymbol{f}_{IMP}$, is as follows:

The Implied Force from the Classical Equations

(1.3.1)$\boldsymbol{f}_{IMP}=q_1\ q_2\boldsymbol{u}/(4\pi\varepsilon_0 r^2)\ +\ [(q_1q_2\mu_0/(4\pi r^2)]\ \boldsymbol{v}_2\ \mathrm{x}\ (\boldsymbol{v}_1\mathrm{x}\ \boldsymbol{u})$

The Fundamental Problem of EM Force Theory

In practice, magnetic and electromotive force experiments generally involve one or more currents. This is due to the fact that the small interactive force between two individual moving charges is hard to measure. As a consequence, reverse engineering is used to find the solution to what is arguably the fundamental problem, which is as follows:

The Fundamental Problem of EM Force Theory

Find the force that a moving charge q_1 exerts on moving a charge q_2 .

Solution Requirements

Consider any arbitrary fixed inertial frame of reference, $\mathbf{IFR}_0$, and two charges, q_1 and q_2. At time t assume the position histories of these two charges are known for all time $\leq t$, or at least for the set of times that are needed to solve the problem. The known variables are therefore q_1, q_2,the positions of these charges over time $\leq t$, and any functions of these variables. Based on this definition, velocities are known variables, but currents are not. It is argued that any proposed solution for the force, $\boldsymbol{f}$, to the fundamental problem should be confined to these known variables, and that it should at least satisfy the following four requirements:

Solution Requirements

- $\boldsymbol{f}$ should be independent of the user's fixed $\mathbf{IFR}_0$.
- $\boldsymbol{f}$ should predict Coulomb's force law when $\boldsymbol{v}_1=\boldsymbol{v}_2$=constant.
- $\boldsymbol{f}$ should explain at least one magnetic experiment.
- $\boldsymbol{f}$ should explain at least one emf experiment.

By way of explanation, not satisfying (a) makes $\boldsymbol{f}$ non-unique, and not satisfying (b) for all $\boldsymbol{v}_1$ makes it clearly wrong. Finally, not satisfying (c) and (d) makes $\boldsymbol{f}$ a speculation.

It is noted that the dependence of $\boldsymbol{f}_{IMP}$ on $\mathbf{IFR}_0$ indicates it does not satisfy requirements (a) and (b), and there is also a problem with (c) because the proof requires the **IFR** to be the stationary wire. Also, (d) is not satisfied because the equation does not explain emf's.

Four Postulates

Suppose at time t charge q_1 is at position $\boldsymbol{r}_0(t)$ and moving at velocity $\boldsymbol{v}_0(t)$, all as measured in an arbitrary $\mathbf{IFR}_0$ Further suppose q_1 emits a ray (i.e., an electric field over an arbitrarily small period of time) at time t which arrives at q_2 at time $t+\Delta t$, all as measured in $\mathbf{IFR}_0$. Let $\mathbf{IFR}(t)$ be defined as a dynamic **IFR** so that the position and velocity of q_1 as measured in $\mathbf{IFR}(t)$ at time t are both zero. Assume the position and velocity of q_2 when the ray arrives there at time $=t+\Delta t$ are $\boldsymbol{r}(t+\Delta t)=r(t+\Delta t)\mathbf{u}(t+\Delta t)$ and $\boldsymbol{v}(t+\Delta t)$, respectively, all as measured in $\mathbf{IFR}(t)$. Define $\boldsymbol{E}_0$ and $\boldsymbol{f}_0$ when the ray hits q_2 at $\boldsymbol{r}(t+\Delta t)$ as the standard field and force values when q_2 is permanently stationary with respect to q_1 in $\mathbf{IFR}(t)$, as follows:

$$\text{(1.6.1)}\quad \boldsymbol{E}_0 = q_1\mathbf{u}(t+\Delta t)/[4\pi\varepsilon_0 r^2(t+\Delta t)]$$

$$\text{(1.6.2)}\quad \boldsymbol{f}_0 = \boldsymbol{E}_0 q_2 = q_1 q_2\mathbf{u}(t+\Delta t)/[4\pi\varepsilon_0 r^2(t+\Delta t)]$$

Four postulates concerning the movement of the electric field through space, its value, $\boldsymbol{E}$, when it arrives at q_2, and the force, $\boldsymbol{f}$,it exerts on q_2 are needed in this work. They are as follows:

$\boldsymbol{\pi}_1$:The electric field moves at c with respect to $\mathbf{IFR}(t)$.

$\boldsymbol{\pi}_2$:At the impact with q_2, $\boldsymbol{E} = \boldsymbol{E}_0$

$\boldsymbol{\pi}_3$:At the impact with q_2, $\boldsymbol{f}$ is collinear with $\boldsymbol{f}_0$.

$\boldsymbol{\pi}_4$:At the impact with q_2, if $\boldsymbol{v}(t+\Delta t)=\mathbf{0}$ then $\boldsymbol{f}=\boldsymbol{f}_0$.

It is argued all four postulates are intuitively reasonable. First, $\boldsymbol{\pi}_1$ is based on the proposition that fields move at c with respect to the source. Second, $\boldsymbol{\pi}_2$ assumes the field emission at t from q_1, which is momentarily stationary in $\mathbf{IFR}(t)$, does not depend on its velocity before and after t, so that Coulomb's electric field law should apply. Third, it is assumed that the force on q_2 is in the direction of the field when it arrives, which is $\boldsymbol{E}_0$. Since $\boldsymbol{f}_0=q_2\boldsymbol{E}_0$, then $\boldsymbol{\pi}_3$ is reasonable. Finally, if $\boldsymbol{v}(t+\Delta t)=\mathbf{0}$ and $\boldsymbol{E}=\boldsymbol{E}_0$, then the force should obey Coulomb's law, so that $\boldsymbol{\pi}_4$ is reasonable.

Road Map

In **Part 2** intuitive reasoning in a series of steps is employed which leads to **(2.4.4)**. The problem with this formula is that it contains a constant ,α, which is undetermined. In **Part 3** a formal proof is offered which shows the equation as derived in **Part 2** is correct, with α=3/2. Then in **Part 4** it is shown this formula correctly explains the result of the double wire magnetic experiment, and similarly in **Part 5** for the emf in a solenoid experiment. In both of these experiments it is shown that α=3/2. When this value of α is plugged into **(2.4.4)**, the final result is obtained in **(2.4.5)**.

Solution Based on Intuitive Methods

Electric Field Law L1

The intuitive analysis of the fundamental problem is based on two laws, $\mathbf{L}_1$ and $\mathbf{L}_2$, which are heavily dependent on the four postulates. The variables involved in $\mathbf{L}_1$ as given below have been previously defined in **Part 1**. This law is actually a restatement of $\boldsymbol{\pi}_2$ in greater detail, which gives the electric field $\boldsymbol{E}(t+\Delta t)$ at its arrival at q_2 at time $t+\Delta t$, as follows:

Electric Field Law L1

$$\text{(2.1.1)}\quad \boldsymbol{E}(t+\Delta t) = q_1\mathbf{u}(t+\Delta t)/[4\pi\varepsilon_0 r^2(t+\Delta t)]$$

Linear Force Law (L2)

This section uses the terms already defined in **Part 1** and law$\mathbf{L_1}$. Suppose the ray emitted by $q_1$at time t moves at velocity c in **IFR**$(t)$and arrives at q_2 at time$t+\Delta t$ with a value of $\boldsymbol{E}(t+\Delta t)$as given by $\mathbf{L_1}$, where $q_2$is at $\boldsymbol{r}(t+\Delta t)$and moving at $\boldsymbol{v}(t+\Delta t)$, all as measured in **IFR**(t). In law$\mathbf{L}_2$cited below the force $\boldsymbol{f}^+$, which is a linear approximation in $v(t+\Delta t)/c$, exerted by the field $\boldsymbol{E}(t+\Delta t)$on q_2 is as follows:

Linear EM Force Law Approximation

(2.2.1)$\boldsymbol{f}^+=q_2\boldsymbol{E}(t+\Delta t)[\,1-\alpha v(t+\Delta t)\cos(\varphi)/c\,]$

The single + superscript indicates the law is of order $O_1(v/c)$. Later on $\boldsymbol{f}^{++}$will be used to indicate the second order solution. The scalar, $v(t+\Delta t)$, is the relative velocity of q_2 in **IFR**(t)at the arrival time, $t+\Delta t$, and φ is the angle between $\boldsymbol{r}(t+\Delta t)$ and $\boldsymbol{v}(t+\Delta t)$ at that instant. Thus, $v(t+\Delta t)\cos(\varphi)$ is the velocity of $q_2$in the direction of $\boldsymbol{r}(t+\Delta t)$, which is the direction of the ray at the point of impact. It will be shown by three different analyses in **Parts 3**, **4**, and **5** that the constant α is given as follows:

(2.2.2)$\alpha = 3/2$

Note that the leading term, $q_2\boldsymbol{E}(t+\Delta t)$, in **(2.2.1)** is the force on q_2 postulated by $\boldsymbol{\pi}_3$if$v(t+\Delta t)=0$ at the instant of impact. The term, $1-\alpha v(t+\Delta t)\cos(\varphi)/c$, is less than unity when $\cos(\varphi)>0$, which is when q_2 is moving away from q_1 at impact, and v.v. when it is moving toward q_1.Intuitively, the idea goes something like this: Suppose a force generated by, say, a spring is exerted on an object moving away from it, so in this case $\cos(\varphi)>0$. Then the force it will exert will be reduced, depending on the relative velocity of the object moving away from the spring (and v.v. if it is moving toward the spring).

This force will be reduced from its normal value down to zero if the object and the spring are moving at the same velocity. Based on this thinking it is argued that**(2.2.1)**is intuitively reasonable in that it should be linearly valid when v/c is small for some positive value α. Also, it will be assumed here and proved in **Part 3** that $\boldsymbol{f}^+\text{->}\mathbf{0}$ when $v(t+\Delta t)\cos(\varphi)\text{->}c$.

Further Analysis of the Linear Force Law

In this section the linear force $\boldsymbol{f}^+$is examined in greater detail and re-written. Inserting $\boldsymbol{E}(t+\Delta t)$from**(2.1.1)** into **(2.2.1)** yields:

(2.3.1)$\boldsymbol{f}^+=\{q_1\boldsymbol{u}(t+\Delta t)/[4\pi\varepsilon_0 r^2(t+\Delta t)]\}$ $\{q_2[1-\alpha\, v(t+\Delta t)\cos(\varphi)/c\,]\}$

The first expression in braces in **(2.3.1)**is the field at q_2, and the second expression is the force this field exerts. It is instructive to simplify this equation. First, define Vas the component of $\boldsymbol{v}(t+\Delta t)$in the direction of $\boldsymbol{r}(t+\Delta t)$, as follows:

(2.3.2)$V = v(t+\Delta t)\cos(\varphi)$

Second, use the formula for $\boldsymbol{f}_0$ as given by **(1.6.2)**. This results in the following alternate formula for the linear force:

Alternate Formulation of the First Order Linear Solution

(2.3.3)$\boldsymbol{f}^+=\boldsymbol{f}_0\ (1-\alpha V/c)$

The force given by **(2.3.1)**,and the equivalent value given by**(2.3.3)**,clearly satisfy the fundamental requirements (a) and (b)in Section **(1.5)**, but not necessarily (c) and (d).These requirements are resolved in **Parts 4** and**5**, respectively. It is noted that an important and intuitively reasonable feature of **(2.3.3)**is that $\boldsymbol{f}^+$isco-linear with$\boldsymbol{f}_0$, which agrees with postulate $\boldsymbol{\pi}_3$.This is not the case with $\boldsymbol{f}_{IMP}$.

The Fundamental Second-Order Solution

As the values of $V/c$are miniscule for the charges involved in the proofs in **Parts 4** and **5**,$\boldsymbol{f}^+$ is used in both analyses. However, in devices such as linear accelerators where it is possible that $V\text{->}c$, a second order term must be added. Based on **(2.3.3)** the general form of the second order solution, $\boldsymbol{f}^{++}$, is given as follows:

Fundamental Second Order Solution

(2.4.1)$\boldsymbol{f}^{++}(V/c) = \boldsymbol{f}_0[1-\alpha\, V/c+\lambda\, V^2/c^2]$

From the discussion in **Section 2.2**, it is proposed that $\boldsymbol{f}^{++}(V/c)\ \text{->}\mathbf{0}$ when $V\text{->}c$. Therefore, the following is assumed:

(2.4.2)$\boldsymbol{f}^{++}(1) = \mathbf{0}$

From **(2.4.1)**and **(2.4.2)**it is clear that $1-(\alpha)(1)+(\lambda)(1)^2=0$, so that:

(2.4.3) $\lambda=\alpha-1$

Thus, on inserting **(2.4.3)** into **(2.4.1)**, the following obtains:

Fundamental Second Order Solution with Unspecified α

(2.4.4) $\boldsymbol{f}^{++}=\boldsymbol{f}_0\ [1-\alpha\, V/c+(\alpha-1)\, V^2/c^2\,]$

Assuming for the moment that $\alpha=3/2$, then **(2.4.4)** becomes:

The Fundamental Second Order Solution with $\alpha=3/2$

(2.4.5) $f^{++} = f_0[1-(3/2)V/c + (1/2)V^2/c^2]$

In (2.4.5)f++is the proposed final solution to the force problem. This solution is simple in appearance and satisfies requirements (a) and (b) as set forth in Section 1.5. The requirements concerning (c) and (d)will be satisfied by the linear analyses in Parts 4 and 5. In Part 3 the entire second order equation is derived, including the requirements that α=3/2 and λ=1/2.Unfortunately, none of these proofs are simple, especially those in Parts 4 and 5 where currents are involved.

Concluding Comments on the Proposed Solution

In conclusion, the EM force as given by f++in(2.4.4), in which the value of α is un resolved, was derived entirely by intuitive methods. Experimental evidence considered in Parts 4 and 5independently shows the linear part of this formula is correct, and that α=3/2. In Part 3 the second order formula given by (2.4.5)is derived analytically, and it is shown that α=3/2 and λ=1/2. It is concluded from this theory and the experimental validations of it that EM forces are entirely the result of electric fields. Magnetic forces are in reality electric field forces which are a result of a dynamic version of Coulomb's law, taking into consideration that fields travel at c with respect to the source and push against charges in a way that depends on relative velocities. As there can be an orthogonal component of the force with respect to the original line of sight, this is mistakenly viewed as a separate magnetic force. It is noted that the pushing force in (2.4.5)approaches zero when V->c, so that charges cannot be accelerated to the velocity of light. Very important, f++is independent of the user's fixed inertial frame of reference. This not the case with the implied classical solution, fIMP .

A Derivation of the Fundamental Solution

Introduction

Theoretical evidence is provided here that the intuitively derived

f^{++} as given by **(2.4.1)** is correct, along with the requirements that

$\alpha=3/2$ and $\lambda=1/2$. This equation is re-written is slightly different format as follows;

(3.1.1) $f^{++}/f_0 = 1-\alpha V/c + \lambda V^2/c^2$

The objective here will be to show **(3.1.1)** is valid and that $\alpha=3/2$ and $\lambda=1/2$. Based on $\pi_3$in **Section 1.6**, it is postulated that f_0 and f^{++} are co-linear with the ray direction as given by $\boldsymbol{r}(t+\Delta t)$. Thus, it is perfectly general to conclude there exists a scalar function β, which satisfies the following:

(3.1.2) $f^{++}/f_0 = (1-\beta)$

The rationale for determining β is based on the following expression:

(3.1.3) $1-\beta = (1-\varepsilon_1)(1-\varepsilon_2)$

As it will be shown that $\varepsilon_1=V/c$ and $\varepsilon_2=(1/2)V/c$, then this implies that:

(3.1.4) $f^{++}/f_0 = 1-\beta = [1-V/c]\,[1-(1/2)V/c]$

$= 1-(3/2)V/c + (1/2)V^2/c^2$

Since **(3.1.4)**agrees with the objective as given by **(3.1.1)**, this completes the proof, along with the requirement that $\alpha=3/2$ and

$\lambda=1/2$. In the following sectionsε_1 and $\varepsilon_2$are determined. While the analysis determiningε_1 is somewhat straightforward, this is not as all the case with ε_2 .

Reduction Factor ε1Due to Lost Momentum

In this section it will be shown that $\varepsilon_1$in **(3.1.3)** is given by $\varepsilon_1=V/c$. Assume a given field $\boldsymbol{E}$hitsq_2, and that the scalar force exerted on this charge isf_0 when $V=0$, where V is given by **(2.3.2)**. Since the electric field has energy, it is assumed this energy can be viewed as having a mass m per unit volume which is moving at velocity c in the direction of $\boldsymbol{r}(t+\Delta t)$ in **IFR**(t), and therefore moving at c-V with respect to q_2. If the cross-sectional area of $q_2$in the direction of $\boldsymbol{E}$is A, then the mass flowing into this charge and stopped by it in time Δt results in a scalar change, Δp, in momentum, which is given as follows:

(3.2.1) $\Delta p = Am(c-V)\Delta t$

From the law of momentum, this change satisfies

(3.2.2) $\Delta p = f_m\Delta t$

In **(2.3.2)**f_m is the resultant force acting on the charge due to the momentum exchange. Thus, from **(3.2.1)** and **(3.2.2)** the following obtains:

(3.2.3) $f_m = Am(c-V)$

If f_0 is the scalar force exerted by the field when $V=0$, then from **(3.2.3)**:

(3.2.4) $f_0 = Amc$

Therefore, from **(3.2.3)** and **(3.2.4)**:

(3.2.5) $f_m/f_0=(c-V)/c=1-V/c$

It is concluded that the fractional loss in force, call it ε_1, due to the total possible momentum not being fully transferred is given as follows:

(3.2.6) $\varepsilon_1= V/c$

This result completes the analysis of ε_1 introduced in **(3.1.3)**. As $\varepsilon_1>0$, this is an indication that the linear force arguments leading up to $\boldsymbol{f}^+$ in **Part 2** have some merit.

Reduction Factor ε2 Due to the Reduced Field Density

In this section it will be shown by somewhat complicate methodology that $\varepsilon_2=(1/2)V/c$ in **(3.1.3)**. The proof employs the following thought experiment: Consider a long railroad track lined end to end with cars of arbitrary length L which move to the right at velocity v. A never-ending constant electric field moving at velocity c with respect to the fixed track travels through the cars, and inside each car is a continuous array of charges which move with it. Therefore, in this experiment $\varphi=0$, $\cos(\varphi)=1$, and $V=v$. It is argued that setting $\varphi=0$ is not an important limitation since V is the only component of $\boldsymbol{v}$ that matters. Further assume that a fixed section of track running from $x=0$ to $x=L$ is marked off, and a car is totally inside that section at $t=0$. Next, define a block of the field of length L, and consider what happens during one cycle when the right edge of the block moves from $x=0$ at $t=0$ to $x=L$ at $t=L/c=T$. At $t=T$ the block is squarely inside the track section. From $\mathbf{L}_1$ and $\mathbf{L}_2$ the relative velocity of any given field block with respect to the interior of any given car is $c-V$. What happens when the block travels a distance L on the fixed track section will now be examined. **Figure 1** below is used to explain the theory.

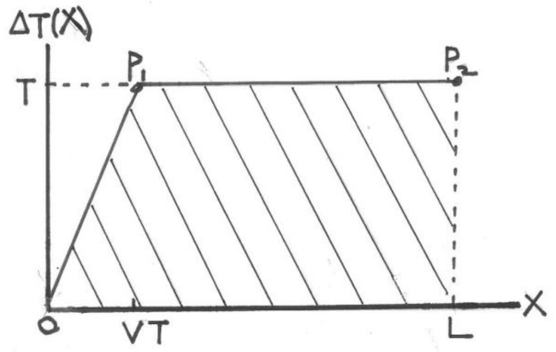

Figure1. *Showing Field Utilization as an Area*

The analysis in this section makes use of the concept of field utilization, $U(V)$, which is shown as the area in the shaded region in **Figure 1** above and is defined as follows:

(3.3.1) $U(V) = \int_0^L \Delta t(x)\, dx$

In **(3.3.1)** $\Delta t(x)$ is the total time at a fixed track position x that the car is present during the cyclic time interval, T, where $T=L/c$. It is noted there is no force at the fixed track position x if the car is not present, and that the car is not always present when $0\le t\le VT$. If the car is not moving, then

$\Delta t(x)=T$ and $U(0)=LT=cT^2$. However, from **Figure 1** it is seen that the amount of time the car is present, as a function of x, is represented by the line $\mathbf{OP_1P_2}$. For $0\le x\le VT$ the value of $\Delta t(x)$ is linearly increasing in x, starting at $x=0$ and ending at $x=VT$. For $x\ge VT$ the car is always present during the cycle. From **(3.3.1)** $U(V)$ is therefore calculated as the total area minus the triangular area given by the closed line, $\mathbf{OTP_1O}$. Since $L=cT$, then:

(3.3.2) $U(V) = LT-(1/2)\, VT^2= (c-V/2)T^2$

Since $U(0)=cT^2$, then **(3.3.2)** can be re-written as:

(3.3.3) $U(V)/U(0) = 1-(1/2)\, V/c$

As it is argued that $U(V)/U(0)$ is the fractional force multiplier due the fact that the car not present on the fixed track section for the entire passage time of the field block. During the absence time, no force is exerted in this section. Calling the lost force factor ε_2, then from **(3.3.3)**:

(3.3.4) $\varepsilon_2 =(1/2)\, V/c$

Conclusion

From **(3.3.4)** and **(3.2.6)** it is concluded that

(3.4.1) $(1-\varepsilon_1)(1-\varepsilon_2) = 1-(3/2)V/c+(1/2)V^2/c^2$

As **(3.4.1)** agrees with the objective given by **(3.1.4)**, this proves that $\boldsymbol{f}^{++}$ as given by **(2.4.5)** is valid, where $\alpha=3/2$ and $\lambda=1/2$. Q.E.D.

Currents In Parallel Wires

First Step in Deriving the 2-Wire Law

In this part the fundamental force law, $\boldsymbol{f}^{++}$, as given by **(2.4.1)** will be employed to explain the outcome of the double-wire magnetic force experiment, which has been noted to obey the Biot-Savart[1] magnetic law. As electron drift velocities are miniscule compared to c, it will turn out that all the velocities in the analysis will have negligible V/c values. Thus, f^{+} in as given by **(2.3.1)** will be used instead of f^{++}. It will be shown that f^{+} explains the results of the experiment, and that $\alpha=3/2$.

The experiment involves sending constant currents, I_1 and I_2, down two parallel wires which are separated by a distance D. In **Figure 2** below the currents I_1 and I_2 are both moving to the left, so that the free electrons flow to the right. The positive charges are assumed to be stationary and equal in number to the free electrons.

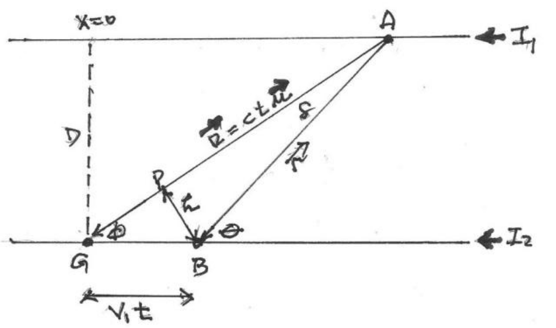

Figure2. *Biot/Savart Two-Wire Experiment*

The Biot-Savart (**BS**) law applied to this problem states that the scalar attractive force f_{BS} exerted by wire #1 on a unit length of wire #2 is:

(4.1.1) $f_{BS} = [\mu_0/(4\pi)]\,[\,2\,I_1 I_2\,/D\,]$

If the currents shown in **Figure 2** are in opposite directions, then $f_{BS}<0$ and the force is repulsive. As f_{BS} in **(4.1.1)** is symmetric in I_1 and I_2, no distinction is made in the equation whether the force is exerted by wire 1 on wire 2 or v.v. In a typical **BS** experiment the current forms a closed loop with $I_1= -I_2$, so in this case the scalar force is repulsive and **(4.1.1)** reduces to the following, where $I=|I_1|$:

(4.1.2) $f = -\,[\,\mu_0/(4\pi)\,]\,[\,2I^2/D\,]$

The objective here in will be to show that **(4.1.1)** can be derived from f^{+} as given by **(2.3.1)**. The method used will be to consider all the rays from the upper wire #1 which arrive at an arbitrary fixed point **B** as shown in **Figure 2** at t=0. If x_1 and x_2 are distances measured on the upper and lower wires, respectively, let $x_2=0$ at point **B**. Of interest is the total force exerted by the rays emitted from all the dx_1 segments of wire #1which arrive at a differential element dx_2 at point **B** at t=0. By necessity these rays are emitted at different times. Unless specifically stated otherwise, the **IFR** will be taken with respect to the wire. Note first that for any wire with current I the total amount of charge, dq, contained in a line segment, dx, is given by:

(4.1.3) $dq=Idt=I(dx/v)$

where dt is the passage time of the current along line segment dx, and v is the drift velocity of the charges in that segment at that instant. In the case shown in **Figure 2** both currents are moving to the left and both x's are increasing to the right, which is the direction of the electron flow.

Now consider dq_1 at point **A** and dq_2 at **B**, where these are the charges contained in dx_1 and dx_2, respectively. In general, these charges may be positive or negative. However, in the special case studied first it is assumed dq_1 is a negative charge moving to the right (opposite to the current I_1) at an average velocity v_1 and dq_2 is a stationary positive charge(v_2=0). This case will be labeled "**MP**", indicating the ray moves from a **M**inus charge in the top wire to a **P**lus charge in the bottom wire. The other cases to be considered are **PM**, **MM**, and **PP**. They are easily solved once the **MP** problem is resolved. According to $\mathbf{L_1}$ the dq_1 charge moving to the right at average velocity v_1 is considered to be at rest, so that the relative movement of dq_2 is to the left at relative velocity v_1(not v_2).Thus, the ray from **A** hits x_2=0 at some point **G**, as shown in the figure, where **G** is the future position of **B** at the time of impact (i.e., at t=0).The vector from **A** to **G** is shown as $\boldsymbol{R}=ct\boldsymbol{u}$, where t is the travel time and $\boldsymbol{u}$ is the unit vector along the line of sight at impact. The initial line of sight when the ray is emitted is shown as $\boldsymbol{r}$ in **Figure 2**, which is at an angle θ to the bottom wire. However, at impact the ray hits the bottom wire at an angle φ. It is seen from the figure that:

(4.1.4) $\varphi = \theta - \delta$

As has previously been described, the situation is similar to a fighter pilot shooting at a moving target, where in this case the actual length R of the shot turns out to be different from the original line-of-sight length r. From f_L the differential force, $df_{\mathbf{MP}}$, exerted by dq_1 on dq_2 in the direction of the line **AG**(i.e.,$\boldsymbol{u}$) in the **MP** case is:

(4.1.5) $df_{\mathbf{MP}} = [dq_1 dq_2/\,(4\pi\varepsilon_0 R^2)]\;\;[1-\,\alpha(v_1/c)\cos(\varphi)]$

Note that it will turn out that the experimental data for this complicated problem involving a virtual infinity of moving charges will require that

$\alpha=3/2$.It is reiterated that the relative velocity of the positive charge at point **B**, as viewed in the **IFR** of dq_1, is moving to the left at velocity v_1 . The angle between $\boldsymbol{u}$ and this relative velocity is

φ, as shown in **Figure 2**. As $\varphi<\pi/2$in the case shown, then$[1-\alpha(v_1/c)\cos(\varphi)]<1$.Thus, there is an $\mathbf{L_2}$force reduction when $\varphi<\pi/2$. As dq_1isnegative, the force as given in **(4.1.5)**is repellent and has an upward component. This is in agreement with the fact that the negative dq_1 charge attracts the positivedq_2charge.If $dF_{\mathbf{MP}}$ is defined as the upward component of df_{MP}, then from **(4.1.5)**:

(4.1.6)$dF_{\mathbf{MP}}=[dq_1dq_2/\ (4\pi\varepsilon_0R^2)]$

$[1-(\alpha v_1/c)\cos(\varphi)]\sin(-\varphi)dq_2$

For the case shown in the figure, $\sin(-\varphi)\geq 0$, $\cos(\varphi)\geq 0$, and $dq_1<0$, so that $dF_{\mathbf{MP}}>0$. The first expression in **(4.1.6)**is the magnitude of the Coulomb electric field given by $\mathbf{L_1}$directed along the line of sight from point **A** to the eventual point of impact, which is at a distance $v_1t$to the left at **G**, relative to the **IFR** of the moving charge q_1.The term given by$[1-(\alpha v_1/c)\cos(\varphi)]$ is the force multiplier factor covered by $\mathbf{L_2}$. The $\sin(-\varphi)$ term is used to obtain the upward force component of the electric field, and dq_2 is used to convert the electric field to a force. As $\sin(-\varphi)= - \sin(\varphi)$, then **(4.1.6)**becomes:

(4.1.7)$dF_{\mathbf{MP}} = -[dq_1dq_2/\ (4\pi\varepsilon_0R^2)]$

$[1-\alpha(v_1/c)\cos(\varphi)]\sin(\varphi)$

In the **MP** case the total upward force $F_{\mathbf{MP}}$ per unit of length along the lower wireis found from**(4.1.7)**by suitably integratingdq_1over the entire upper wire and dq_2 over a unit distance,$\Delta x_2=1$. Thus, in general terms, without specifying the limits on the integrals:

(4.1.8)$F_{\mathbf{MP}} =-\iint[dq_1dq_2/\ (4\pi\varepsilon_0R^2)]\ [1-\alpha(v_1/c)\cos(\varphi)]\sin(\varphi)$

As all the dq_2's along the lower wire have the same upward force exerted on them, the contribution to F_{MP} from the dq_2portion of the integration is found by replacingdq_2withthe total Δq_2 charge along a length of $\Delta x_2=1$. From **(4.1.1)**$\Delta q=I_2\Delta t$, where $\Delta t=\Delta x_2/v=1/v$. In this case the velocity of the upper negative charge is v_1. Thus, the relative velocity of the stationary lower positive charge, with respect to the upper charge, is $v=v_1$. Therefore:

(4.1.9)$\Delta q_2= I_2|\Delta x_2|/v= I_2/v_1$

Inserting **(4.1.9)**into **(4.1.8)** and neglecting the limits yields the following:

(4.1.10)$F_{\mathbf{MP}}=-[I_2/(4\pi\varepsilon_0v_1)]\ \int\ dq_1\sin(\varphi)[1-\alpha(v_1/c)\cos(\varphi)]/R^2$

It is convenient to use x as the general position indicator for both wires, where $x=0$ at **B**, and to integrate with respect to x. Then, from **(4.1.10)**:

(4.1.11)$F_{\mathbf{MP}}=-[I_2/(4\pi\varepsilon_0v_1)]$

$\int_{-\infty}^{\infty}\ dx[dq_1/dx]\sin(\varphi)[1-\alpha(v_1/c)\cos(\varphi)]/R^2$

From **(4.1.2)** it is noted that $dq_1/dx= I_1/v_1$. Thus,**(4.1.11)**becomes:

(4.1.12)$F_{\mathbf{MP}} = -[I_1I_2/(4\pi\varepsilon_0v_1^2)]$

$\int_{-\infty}^{\infty}\ dx\ \sin(\varphi)\ [1-\alpha(v_1/c)\cos(\varphi)]/R^2$

Since $x=r\cos(\theta)$ and $D/r=\sin(\theta)$, then $x=D\mathrm{ctn}(\theta)$ and

$dx = -[D/\sin^2(\theta)]d\theta$. As the integral given by **(4.1.12)** involves

$-\infty\leq x\leq\infty$, this implies $\pi\leq\theta\leq 0$. Accordingly, on inverting the upper and lower limits so that $0\leq\theta\leq\pi$,and noting that $D/R=\sin(\varphi)$, **(4.1.12)**can be rewritten as:

(4.1.13)$F_{\mathbf{MP}} = [I_1I_2/(4\pi\varepsilon_0v_1^2D)]$

$\int_0^{\pi} d\theta\sin^3(\varphi)[1-\alpha(v_1/c)\cos(\varphi)]/\sin^2(\theta)$

From **(4.1.13)** it is seen that $F_{\mathbf{MP}}>0$, which is correct since the **MP** force is attractive. In the next section $F_{\mathbf{MP}}$ will be evaluated from **(4.1.13)**. Then from this analysis it will be a simple matter to find the total scalar Biot-Savart upward force, F_{BS}, as given by **(4.1.1)**.

Evaluation of FMP Given by (4.1.13)

In this section the scalar $F_{\mathbf{MP}}$will be evaluated from **(4.1.13)**. To this end it is convenient to define K_1 and K_2 as follows:

(4.2.1)$K_1 = [I_1I_2/(4\pi\varepsilon_0v_1^2D)]$

(4.2.2)$K_2 = \sin^3(\varphi)\ [1-\alpha z_1\cos(\varphi)]\ /\ \sin^2(\theta)$

Where

(4.2.3)$z_1=v_1/c$

Inserting **(4.2.1)** and **(4.2.2)** into **(4.1.13)** yields:

(4.2.4)$F_{\mathbf{MP}} = K_1\int_0^{\pi}\ K_2d\theta$

Note from **(4.1.4)** that $\varphi = \theta - \delta$. Thus, from **(4.2.2)**:

(4.2.5)$K_2 = \sin^3(\theta - \delta)\ [1-\alpha z_1\cos(\theta - \delta)]\ /\ \sin^2(\theta)$

From **(4.2.2)** and **(4.2.3)** it is seen that K_2 is a dimensionless quantity. Accordingly, the integral in **(4.2.4)** is a routine mathematical problem which is evaluated in **Appendix 1**. The result is:

(4.2.6) $F_{MP} = K_1 (2 + z_1^2)$

Replacing z_1 by v_1/c and K_1 by $[I_1I_2/(4\pi\varepsilon_0 v_1^2 D)]$ in **(4.2.6)** yields:

(4.2.7) $F_{MP} = [I_1I_2/(4\pi\varepsilon_0 v_1^2 D)][2 + v_1^2/c^2]$

This is the final result for F_{MP}. It will be used in the next section.

Total Force per Unit of Length

It is noted that F_{MP} is the total upward force per unit of length along the lower wire due to the **MP** charges. The total upward scalar force F_{tot} per unit length of the lower wire is:

(4.3.1) $F_{tot}=F_{MP} + F_{PM} + F_{MM} + F_{PP}$

F_{PM} is the force per unit of length along the lower wire created by a stationary positive charges in the upper wire acting on negative charges in the lower wire moving to the right(see **Figure 2**) with velocity v_2. The analysis of this problem is virtually identical to the F_{MP} problem. Instead of point **G** being marked off to the left of **B** at a distance v_1t, **G*** is marked off to the right of **B** at a distance v_2t^*. If the figure is viewed from the reverse side, the problem is identical to the F_{MP} case, except the roles of v_1 and v_2 are reversed. Thus, from **(4.2.7)** and on reversing v_1 and v_2 :

(4.3.2) $F_{PM} = [I_1I_2/(4\pi\varepsilon_0 v_2^2 D)] [2 + v_2^2/c^2]$

Next, note that both F_{MM} and F_{PP} involve charges which are stationary with respect to each other. Therefore, in this case there is no v_1^2/c^2 term in **(4.2.7)** and no v_2^2/c^2 term in **(4.3.2)**. As the forces are repulsive, then:

(4.3.3) $F_{MM} = F_{PP} = -2\, I_1I_2/(4\pi\varepsilon_0 v_1 v_2 D)$

Accordingly, the result given by **(4.3.1)** is, after a little algebra:

(4.3.4) $F_{tot} = 2\, [I_1I_2/(4\pi\varepsilon_0 D c^2)]$

This result is somewhat similar to the Biot-Savart formula given by **(4.1.1)**. In the next section it will be shown that **(4.3.4)** is in fact this formula.

Note that in the special case where the same current flows around a loop, then $I_2= - I_1$. In this situation, define $I=|I_1|$. The following formula therefore obtains for the scalar force, f :

(4.3.5) $f= - 2\, I^2/ (4\pi\varepsilon_0 D c^2)$

As f<0, the force given by **(4.3.5)** is repulsive.

Conclusion of Proof and Finding Maxwell's c

It is contended that F_{tot} as given by **(4.3.4)** and f_{BS} as given by **(4.1.4)** are equal forces. It is seen this will be the case if c satisfies the following:

(4.4.1) $c = 1 /\sqrt{(\varepsilon_0\mu_0)}$

As **(4.4.1)** has been experimentally verified, then $F_{tot}=f_{BS}$ and there fore the linear solution given by f^+ correctly explains the Biot-Savart magnetic force law as applied to the double wire experiment. As a bonus, Maxwell's formula for the velocity of light is also a bi-product. Q.E.D.

It is interesting that the formula for c as given by **(4.4.1)** was arrived at by Maxwell using classical **EM** theory, in spite of the fact that f^{++} indicates magnetic forces are actually electric field forces. However, since the definition of the ampere and μ_0 areintertwinedby the results of the parallel wire experiment when $I_1= - I_2$,it turns out that μ_0 can be set arbitrarily, which actually is the case. Thus, **(4.4.1)** is meaningful even though magnetic forces do not exist.

Comment on the Absence of Drift Velocities in the Formula

It is strange that drift velocities play no role in the formula for the repulsive force in the double wire experiment, even though magnetic forces as given by classical theory and the forces postulated in this work should presumably depend on them. Out of curiosity my colleague, David Banks, and I performed a series of experiments to see if the repulsive force was indeed independent of the two drift velocities. These experiments maintained $I_1=-I_2$ and then compared the repulsive force when $v_1=v_2$ with situations where $v_1 \neq v_2$, keeping all the other variables the same. We found the repulsive forces were in fact independent of the drift velocities.

FARADAY'S LAW

Introduction

In **Part 4** the linear solution, f^+, with the requirement that $\alpha=3/2$, was shown to explain the magnetic force in the parallel wire experiment. In this analysis f^+ will also be shown by a somewhat involved mathematical analysis that it also explains Faraday's law concerning emf's. The problem, once again, is that there is a virtual infinity of charges in currents, and f^+ is a law which concerns them only in pairs. As it is argued the primary application of Faraday's law involves circular currents, then emf's will be investigated only in this situation. Specifically, the current in a solenoid will be singled out. Attention is drawn to **Figure 3** below, which depicts two rings in a circular coil of radius *r*. The two rings shown are separated in height by a distance of *Z*, which is the length of the line $\mathbf{P}_1\mathbf{P}_2$. Not shown is the total length $\mathcal{L}$, which is assumed to be sufficiently large that the conditions at the ends may be neglected (as is the case with the Faraday's law calculations for coils). The solenoid has *n* rings/meter, and it will be approximated by an infinite number of "differential rings", in which there are *ndZ*rings in a vertical height *dZ*. The lower ring depicted centered at $\mathbf{P}_0$is assumed to be at the center of the coil length. Ultimately, the circular force on this ring at $\mathbf{P}_1$ will be ascertained, which will in turn lead to a calculation of the emf. As a continuous charge model will be employed concerning the ring current, there is a problem using laws $\mathbf{L}_1$ and $\mathbf{L}_2$ in that the electric forces approach infinity when nearby charges are considered. The technique used to circumvent this problem is to consider the force at $\mathbf{P}_4$ in **Figure3** below, where $\mathbf{P}_4$is at $\boldsymbol{X}=X\boldsymbol{i}$ and $X\text{->}0$. Suppose this force turns out to be proportional to *X*. Then the force at the ring at $\mathbf{P}_1$ is found by extrapolating by setting $X=1$. From Faraday's law the emf (call it K^*) satisfies:

(5.1.1)$K^* = \text{emf} = -\, n^2\mu_0\mathcal{L}A dI/dt$

Where $A=\pi r^2$.**Figure 3** is as follows:

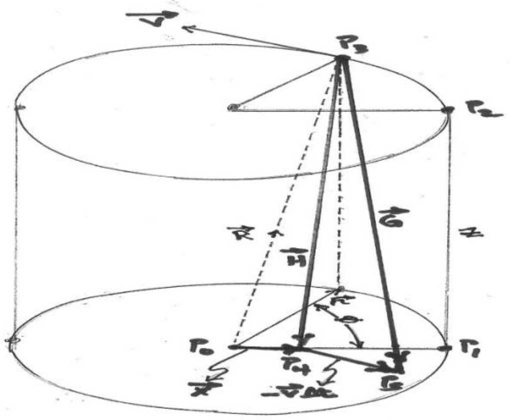

Figure3. *Solenoid used in faraday analysis*

It is reiterated that certain approximations are made in deriving **(5.1.1)** from Faraday's law, one of which is neglecting the difficulties arising near the ends of the solenoid. In using $\mathbf{L}_1$ and $\mathbf{L}_2$ to derive the emf, somewhat similar approximations are made. Chiefly among these are (a) assuming a small *X*as discussed above, (b) neglecting higher order terms in *v/c*, and (c) only examining the emf at the half-way point in the solenoid.

Drift Velocity and Current

The solenoid shown in **Figure 3** consists of rings where the angle θ is measured with respect to arbitrary point $\mathbf{P}_1$. The vertical distance *Z* is measured upward from $\mathbf{P}_1$. The line $\mathbf{P}_1\mathbf{P}_2$extends vertically from the lower ring to the upper ring. From the figure:

(5.2.1)$\boldsymbol{r}(\theta)= r\,[\boldsymbol{i}\cos(\theta) + \boldsymbol{j}\sin(\theta)\,]$

It is reiterated that approximation signs are omitted in this work when higher order terms are eliminated. Now, let$\boldsymbol{V}(\theta,Z,t)$ be the drift velocity at time *t*at an arbitrary pointon a ring defined by (θ,Z).Define$V_0=V(0,0,0)$as the drift velocity at $\mathbf{P}_1$ at$t=0$. It is assumed the drift velocity is very small and both it and the current are the same everywhere in the solenoid. Thus, it is assumed that $V(\theta,Z,t)=V(0,0,t)$, so that

(5.2.2)$\boldsymbol{V}(\theta,Z,t)= V(0,0,t)\;[\,-\boldsymbol{i}\sin(\theta)+ \boldsymbol{j}\cos(\theta)\,]$

The following linear approximation is made concerning in**(5.2.2)**:

(5.2.3)$V(0,0,t) = V_0\,(1+at)$

Also, if $I(\theta,Z,t)$ is the current and if I_0 is defined as $I_0=I(0,0,0)$, then on assuming the current is proportional to the drift velocity:

(5.2.4)$I(\theta,Z,t) = I_0V/V_0 = I_0\,(1+at)$

Evaluating Certain Variables Used in Section 5.4

In this section certain important variables will be defined and evaluated which will ultimately play a role in **Section5.4**. From **Figure 3** the vector $\boldsymbol{H}$ runs from point $\mathbf{P}_3$ to $\boldsymbol{X}$at $\mathbf{P}_4$. It is seen that $\boldsymbol{H}+(\boldsymbol{r}-\boldsymbol{X})+Z\boldsymbol{k}=\mathbf{0}$, where $\boldsymbol{r}$ is given by **(5.2.1)**. Thus:

(5.3.1)$\boldsymbol{H} = \boldsymbol{i}\,[X - r\cos(\theta)] - \boldsymbol{j}r\sin(\theta) - \boldsymbol{k}Z$

Normalizing all variables in **(5.3.1)**by dividing by *r* yields

(5.3.2)$\boldsymbol{h}= \boldsymbol{H}/r = \boldsymbol{i}(x\text{-}\cos(\theta)) + \boldsymbol{j}\sin(\theta) - \boldsymbol{k}z$

where $x=X/r$and$z=Z/r$. From **(5.3.2)**$h^2=(x\text{-}\cos(\theta))^2+\sin(\theta)^2+1+z^2$. Thus:

(5.3.3)$h^2 = 1 + x^2 - 2\,x\cos(\theta) + z^2$

Note that $\boldsymbol{V}$ is tangent to the upper ring at point $\mathbf{P}_3$in **Figure 3**. Therefore, from law $\mathbf{L}_1$ the relative movement of a stationary test charge *Q* at $\mathbf{P}_4$ is $-\boldsymbol{V}$. Now suppose the ray emitted at point $\mathbf{P}_3$ travels along the vector $\boldsymbol{G}$and arrives at $\mathbf{P}_5$ at time=0. Since$\mathbf{P}_5$ is at $X\boldsymbol{i}\text{-}\boldsymbol{V}\Delta t$, then from **(5.3.1)**:

(5.3.4) $\boldsymbol{G} = \boldsymbol{H} - \boldsymbol{V}\Delta t = \boldsymbol{i}(X - r\cos(\theta)) - \boldsymbol{j}r\sin(\theta) - \boldsymbol{k}Z - \boldsymbol{V}\Delta t$

Dividing the terms in **(5.3.4)** by r yields:

(5.3.5) $\boldsymbol{g}=\boldsymbol{G}/r=[x-\cos(\theta)+(V\Delta t/r)\sin(\theta)]\boldsymbol{i} - [\sin(\theta)+(V\Delta t/r)\cos(\theta)]\,\mathbf{j}-z\mathbf{k}$

After a little algebra, **(5.3.5)** implies the following:

(5.3.6) $g^2=1+x^2-2x\cos(\theta)+z^2+(V\Delta t/r)^2+2x(V\Delta t/r)\sin(\theta)$

From **(5.3.6)** and **(5.3.3)**:

(5.3.7) $g^2 - h^2 = (V\Delta t/r)^2 + 2x(V\Delta t/r)\sin(\theta)$

From **Figure 3** and the law of cosines, $H^2$is given as follows:

(5.3.8) $H^2 = G^2 + (V\Delta t)^2 - 2(V\Delta t)G\cos(\varphi)$

Solving **(5.3.8)** for $\cos(\varphi)$ yields

(5.3.9) $\cos(\varphi) = [G^2 - H^2 + (V\Delta t)^2] / [2\,GV\Delta t]$

Dividing the both numerator and denominator in **(5.3.9)** by r^2 yields:

(5.3.10) $\cos(\varphi) = [g^2 - h^2 + (V\Delta t/r)^2] / [2\,g\,(V\Delta t/r)]$

Inserting **(5.3.7)** into **(5.3.10)** yields, after a little algebra:

(5.3.11) $\cos(\varphi) = [x\sin(\theta) + V\Delta t/r] / g$

Dropping the O_1 term in **(5.3.11)** yields

(5.3.12) $\cos(\varphi) = x\sin(\theta)/g$

Finally, if x grows small, then $g\text{->}h$ and $h\text{->}(1+z^2)^{1/2}$. Thus, from **(5.3.12)**:

(5.3.13) $\cos(\varphi) = x\sin(\theta)/(1+z^2)^{1/2}$

Note from **(5.4.2)** that dq at $\mathbf{P}_3$ is given by

(5.3.14) $dq = (dq/dt)\,(dt/dx)\,rd\theta = Ird\theta/V$

Since at $\mathbf{P}_3$ at time= $-\Delta t$the current is $I_0(1-a\Delta t)$ and $V=V_0(1-a\Delta t)$, then

(5.3.15) $dq= I_0(1-a\Delta t)rd\theta/[V_0(1-a\Delta t)] = I_0\,rd\theta/V_0$

Equation **(5.3.15)** is the primary result of this section.

Derivation of the Force at P4

In this section the force $\boldsymbol{F}$ at point$\mathbf{P}_4$shown in **Figure 3**(i.e., at $\boldsymbol{X}=X\boldsymbol{i}$)due to the current in the particular coil will be determined, where $X\text{->}0$. Also, in the final analysis, only the force orthogonal to the radius at $\mathbf{P}_4$will ultimately enter into the calculation for the solenoid emf. As it is in the $\boldsymbol{j}$ direction at $X\boldsymbol{i}$ in the figure, this force component will be called F_j. The calculations will consider all the rays from the entire solenoidhitting at $\mathbf{P}_4$ at time=0. A typical emission point is shown in **Figure 3**as$\mathbf{P}_3$. If **IFR***(0)* is the **IFR** with respect to the current velocity at time=0 at point $\mathbf{P}_3$ and it is labeled **IFR(P$_3$)**, the ray from $\mathbf{P}_3$ moves to$\mathbf{P}_5$, which is the future position of $\mathbf{P}_4$lying along the vector $\boldsymbol{G}$, as shown in the figure. The passage time, Δt, and the other variables shown have all been found in **Section** 5.3. In a manner similar to **Part 4** the analysis finds $\boldsymbol{F}=\boldsymbol{F}_{MP}+\boldsymbol{F}_{PP}$, where $\boldsymbol{F}_{MP}$ and $\boldsymbol{F}_{PP}$ are the vector forces on a positive test charge Q at $\mathbf{P}_4$ (i.e., at $\boldsymbol{X}$). $\boldsymbol{F}_{MP}$ is the force on Q due to the minus charges moving through the arc given by $rd\theta$ at $\mathbf{P}_3$.Conversely,$\boldsymbol{F}_{PP}$ is the force due to the positive stationary charges situated there. From laws $\mathbf{L}_1$ and $\mathbf{L}_2$, $\boldsymbol{F}_{MP}$ is given as follows:

$$\textbf{(5.4.1)}\ \boldsymbol{F}_{MP}=[Q/(4\pi\varepsilon_0)]\int_{-\frac{\mathscr{L}}{2}}^{\frac{\mathscr{L}}{2}} n dZ \int_0^{2\pi} d\theta(dq/d\theta)(\boldsymbol{G}/G^3)[1-(\alpha V/c)\cos(\varphi)]$$

The terms in **(5.4.1)**given by $[Q/(4\pi\varepsilon_0)](\boldsymbol{G}/G^3)[1-(\alpha\ V/c)\cos(\varphi)]$ represent the force on the test charge Q at $\mathbf{P}_5$ in **IFR(P$_3$)** due to the moving charges at $\mathbf{P}_3$. The amount of charge at $\mathbf{P}_3$in the interval covered by $d\theta$ is $(dq/d\theta)d\theta$. From**(5.3.15)**, $dq=I_0rd\theta/V_0$. Rather than summing over the $n\mathscr{L}$ individual rings in **(5.4.1)**, an approximation is made which assumes there are ndZ rings in the interval between Z and $Z+dZ$. Setting $1/\varepsilon_0=\mu_0c^2$, $Z=rz$,$\boldsymbol{G}=r\boldsymbol{g}$ and approximating $\mathscr{L}/2$ with ∞yields the following:

$$\textbf{(5.4.2)}\ \boldsymbol{F}_{MP} = [Q\mu_0nc^2I_0/(4\pi)] \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta(\boldsymbol{g}/g^3)\,[1-(\alpha V/c)\cos(\varphi)]/V_0$$

Since $X\text{->}0$, then $g\text{->}h\text{->}\sqrt{(1+z^2)}$. Also, $\Delta t\text{->}hr/c$, $V\text{->}V_0(1-ahr/c)$, and $\cos(\varphi)\text{->}x\sin(\theta)/h$. Then

$[1-(\alpha/c)V\cos(\varphi)]=[1-(\alpha/c)V_0(1-ahr/c)x\ \sin(\theta)/h]$. In turn, this expression can be re-written as$[1-\alpha V_0/c+\alpha a V_0 rx\sin(\theta)/c^2]$. Thus,**(5.4.2)** can be re-written as

$$\textbf{(5.4.3)}\ \boldsymbol{F}_{MP}=[Q\mu_0nc^2I_0/4\pi] \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta(\boldsymbol{g}/g^3)\,[1-\alpha V_0/c+\alpha a V_0 rx\sin(\theta)/c^2]/V_0$$

Note that $\boldsymbol{F}_{PP}$ can be found from **(5.4.3)** by replacing $\boldsymbol{g}$ with $-\boldsymbol{g}$ and setting a=0.The result is as follows:

(5.4.4) $\boldsymbol{F}_{PP} = -[Q\mu_0 nc^2 I_0/(4\pi)]$

$\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta[\boldsymbol{g}/g^3]\,[1-\alpha V_0/c)]/V_0$

As the total vector force, $\boldsymbol{F}$, satisfies $\boldsymbol{F}=\boldsymbol{F}_{MP}+\boldsymbol{F}_{PP}$, and as $g=h=(1+z^2)^{1/2}$, then from **(5.4.3)** and **(5.4.4)**:

(5.4.5) $\boldsymbol{F}=[Q\mu_0 nc^2 I_0/(4\pi)]\int_{-\infty}^{\infty} dz\int_0^{2\pi} d\theta(\boldsymbol{g}/g^3)\alpha arx \sin(\theta)/c^2$

As g_j-> $-\sin(\theta)$ and g->h->$(1+z^2)^{1/2}$, then from **(5.4.5)** and a little algebra the scalar component F_j can be found as follows:

(5.4.6) $F_j=[Q\mu_0 nI_0 arx\alpha/(4\pi)]\int_{-\infty}^{\infty} dz/(1+z^2)^{3/2}\int_0^{2\pi} d\theta \sin(\theta)^2$

The two integrations in **(5.4.6)** are routinely found as 2 and π, respectively. Thus, F_j in **(5.4.6)** becomes:

(5.4.7) $F_j= -Qn\mu_0(I_0 a)\, rx\alpha/2$

Finding the Solenoid emf from Fj

As the point **P**$_1$ in **Figure 3** is arbitrary, choosing any other point, say **P**$_3$, as the basis of measuring θ would be equally valid. Thus, F_j is the orthogonal force all the way around the lower ring. The emf (call it K) due to the moving charges can therefore be found as follows:

(5.5.1) $K = (2\pi r)(n\mathcal{L})(F_j/Q) = -n^2\mu_0\mathcal{L}\,(I_0 a)\,x\alpha\pi r^2$

As $I_0 a=dI/dt$ and the ring area is $A=\pi r^2$, then from **(5.5.1)**:

(5.5.2) $K = -n^2\mu_0\,\mathcal{L}\,A\alpha dI/dt$

From **(5.1.1)** K^*=emf $= -n^2\mu_0\mathcal{L} AdI/dt$. Thus, from **(5.5.2)**:

(5.5.3) $K= \alpha K^* = (3/2)\, K^*$

Note that α in **(5.5.3)** has been set at 3/2, in keeping with the results of **Parts 3** and **4**. This value will be assumed here, and it will be investigated as to where it leads. From **(5.5.3)** it is seen that the effect of the force due to the current contributes more to the emf than is measured by experiment. The reason for this is that there is a time-varying field present in the solenoid which creates the current. Assume the field across the particular coil at the time considered in the analysis (i.e, at time=0) is $\boldsymbol{E}$. Let D be the total distance around the entire coil (i.e., $D=2\pi rn\mathcal{L}$) and d be the distance to an arbitrary point along this path. The scalar field, $E(d)$, linearly decreases from E to 0 as d runs from 0 to D. The average scalar value of the field is therefore $E/2$. Thus, there is an energy gain of $K^*/2$ in moving a sample charge from d=0 to $d=D$. This gain must be subtracted from the value of K found in **(5.5.3)** and **(5.1.1)**. Thus,

(5.5.4) emf $= K - K^*/2 = (3/2)\,K^* - K^* = -n^2\mu_0\mathcal{L}\,A(dI/dt)$

Equation **(5.5.4)** completes the proof that Faraday's law as applied to a solenoid is predicted by $\boldsymbol{f}^{\;+}$, where in this case α=3/2. Q.E.D.

Results And Conclusions

It is argued in this work that **EM** laws concerning currents are not fundamental. These laws are relationships found from experiments, and they should be derivable from a single fundamental law which gives the force, $\boldsymbol{f}^{++}$, that a single moving charge q_1 exerts on a single moving charge q_2 .In actual practice reverse engineering is employed to infer a classical solution to this two charge problem, and it is shown this solution is unsatisfactory. The proposed solution offered in this work has several important features, as follows:

- it is independent of the user's fixed inertial frame of reference
- it is experimentally verified in a magnetic experiment
- it is experimentally verified in an emf experiment
- except for the value of α, it can be intuitively derived
- it is satisfied by a theoretical proof
- $\boldsymbol{f}^{++}$ ->**0** when V->c

The most important finding in this work is that $\boldsymbol{f}^{++}$ is entirely due to electric fields; magnetic forces do not exist, per se. Since a ray emanating from q_1 hits q_2 at a later time, there can be a force component which is orthogonal to the original line of sight that has nothing to do with a special magnetic force. Though Maxwell's equations concerning **EM** forces and his formula for c are viewed as example problems, they are nevertheless very useful analytical tools.

Appendix A – FMP Proof

In this appendix the integral portion of **(4.2.4)** will be evaluated. From **Figure 2** and **L**$_1$ it is seen that $R=ct$, where t is the passage time of the light ray from point **A** to **G** (where **G** is the future position of point **B** in the **IFR** of the moving electron at **A**). The distance moved by the positive charge during t is v_1t, as shown by the line **BG** in the figure. For notational convenience equality signs are used rather than approximation signs in equations after the $O_3(z_1)$ terms have been omitted. As δ is small, then $\sin(\delta)$ and $\cos(\delta)$ are approximated by $\sin(\delta)=\delta$ and $\cos(\delta)=1-\delta^2/2$. Note from the figure that h is the distance along the line **BP**, which is orthogonal to **AG**. Then, on dropping higher order terms in δ, h is evaluated as

$h=r\sin(\delta)=D\sin(\delta)/\sin(\theta)=D\delta/\sin(\theta)$. Also,

$h=v_1t\sin(\varphi)=v_1t\sin(\theta\ \delta)$.Thus,$D\delta/\sin(\theta)=v_1t\sin(\theta-\delta)$, and therefore $t=D\delta/[v_1\sin(\theta)\sin(\theta-\delta)]$.Also from **Figure 2** it is seen that $ct\sin(\varphi)=D$. Thus, $\delta=(v_1/c)\sin(\theta)=z_1\sin(\theta)$. Note from trigonometry that$\sin(\theta-\delta)=\sin(\theta)\cos(\delta)-\cos(\theta)\sin(\delta)$. Also,

$\cos(\theta-\delta)=\cos(\theta)\cos(\delta)+\sin(\theta)\sin(\delta)$and

$\sin(\theta-\delta)=(1-\delta^2/2)\sin(\theta)-z_1\sin(\theta)\cos(\theta)$.

Then after all approximation signs have been dropped,

$\cos(\theta-\delta)=(1-\delta^2/2)\cos(\theta)+\delta\sin(\theta)$.From **(4.2.5)**

$K_2=\sin^3(\theta-\delta)[1-\alpha z_1\cos(\theta-\delta)]/\sin^2(\theta)$. Thus,

$K_2=\{(1-\delta^2/2)\sin(\theta)$ $z_1\sin(\theta)\cos(\theta)\}^3$.Since$\delta=z_1\sin(\theta)$, then factoring $\sin(\theta)$ in the first braces inthis equation for K_2yields

$K_2=\sin(\theta)\{1-z_1^2\sin^2/2-z_1\cos\}^3$

$\{1-\alpha z_1[(1-\delta^2/2)\cos(\theta)+z_1\sin(\theta)^2]\}$.

This equation is of the following form:

(A.1)$K_2=\sin(\theta)\,\beta_1\beta_2$

where β_1 andβ_2 are (after dropping the O_2terms in δ):

$\beta_1=\{1-z_1^2\sin(\theta)^2/2-z_1\cos(\theta)\}^3$ and

$\beta_2=\{1-\alpha z_1\cos(\theta)-\alpha z_1^2\sin(\theta)^2\}$.

Expanding the $\beta_1$cubic yields, after eliminating the O_3 terms in z_1,

$\beta_1=1-3z_1\cos(\theta)-3z_1^2\sin(\theta)^2/2+3z_1^2\cos(\theta)^2$.

After eliminating higher order terms,$\beta_1\beta_2$ becomes

(A.2)$\beta_1\beta_2=1-\alpha z_1\cos(\theta)-\alpha z_1^2\sin(\theta)^2-3z_1\cos(\theta)+3\alpha z_1^2\cos(\theta)^2-3z_1^2\sin(\theta)^2/2+3z_1^2\cos(\theta)^2$

Note from from **(4.2.4)**and **(A.1)**that

(A.3)$F_{MP}=K_1\int_0^{\pi}\sin(\theta)\beta_1\beta d\theta$

Since $\alpha=3/2$, the integration in **(A.3)**routinely yields (after eliminating the O_3 terms):

(A.4)$F_{MP}=K_1[2+z_1^2]$

This is the result used in **Section 4.2**. Q.E.D.

References

[1] Biot, J. and Savart, F. (1820), see, e.g.,Physics for Scientists and Engineers, by Tipler and Mosca.

[2] Einstein, A. (1905), ”ZurElektrodynamikbewegterKorper”, Annalen der Physik,17, 891.

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