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Polarization Density Fractal Dimension for Characterizing Shajara Reservoirs of the Permo-Carboniferous Shajara Formation

Khalid Elyas Mohamed Elameen Alkhidir, Ph. D*

Department of Petroleum and Natural Gas Engineering, College of Engineering King Saud University, Saudi Arabia

***Corresponding Author:** Prof. Khalid Elyas Mohamed Elameen Alkhidir, Ph. D Department of Petroleum and Natural Gas Engineering, College of Engineering King Saud University, Saudi Arabia, Email: kalkhidir@ksu.edu.sa*

ABSTRACT

The quality and assessment of a reservoir can be documented in details by the application of polarization density. This research aims to calculate fractal dimension from the relationship among polarization density, maximum polarization density and water saturation and to confirm it by the fractal dimension derived from the relationship among capillary pressure and water saturation. In this research, porosity was measured on real collected sandstone samples and permeability was calculated theoretically from capillary pressure profile measured by mercury intrusion techniques. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, polarization density, and maximum polarization density and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between polarization density and maximum polarization density versus logarithm wetting phase saturation. The slope of the first procedure is positive = 3- Df (fractal dimension). The second procedure for obtaining the fractal dimension was completed by plotting logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure is negative = Df -3. On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units. These units from bottom to top are: Lower Shajara polarization density Fractal Dimension Unit, Middle Shajara polarization density Fractal Dimension Unit, and Upper Shajara polarization density Fractal Dimension Unit. The three reservoir units were also confirmed by capillary pressure fractal dimension. It was found that the obtained fractal dimension increases with increasing grain size and permeability.

Keywords: *Shajara Reservoirs; Shajara Formation; polarization density fractal dimension; capillary pressure fractal dimension.*

INTRODUCTION

[1] Reported that the wetting phase saturation can be described as function of capillary pressure and fractal dimension. [2] Demonstrated that the Purcell model was found to be the best fit to the experimental data of the wetting phase relative permeability for the cases as long as the measured capillary pressure curve had the same residual saturation as the relative permeability curve.

They also reported that in the reverse procedure, capillary pressure could also be computed once relative permeability data are available. [3]Derived theoretically a model to correlate capillary pressure and resistivity index based on the fractal scaling theory. Their results demonstrated that the model could match the experimental data in a specific range of low water saturation. [4] Showed the fractal dimension resulting from longer transverse NMR relaxation times and lower capillary pressure reflects the volume dimension of larger pores. They also reported that the fractal dimension derived from the short NMR relaxation times is similar to the fractal dimension of the internal surface. [5] Reported that the fractal dimensions can be used to represent the complexity degree and heterogeneity of pore structure, and the coexistence of dissolution pores and large inter granular pores of Donghetang sandstones contributes to a heterogeneous pore throat distribution and a high value of fractal dimension. [6] Studied the relationship among capillary pressure (PC), nuclear magnetic transverse relaxation time (T2) and resistivity index (I). An increase of bubble pressure fractal dimension and pressure head fractal dimension and decreasing pore size distribution index and fitting parameters m*n due to possibility of having interconnected channels was confirmed. [7,8]An increase of fractal dimension with increasing arithmetic, geometric relaxation time of induced polarization, permeability and grain size was investigated by [9,10,11]. An increase of seismo electric and resistivity fractal dimensions with increasing permeability and grain size was described [12, 13, and 14]. An increase of electro kinetic fractal dimension and electric power fractal dimension of nuclear magnetic resonance with increasing permeability and grain size was reported by [15].

Material and Method

Samples were collected from the surface type section of the Shajara reservoirs of the Permo-carboniferous Shajara formation at latitude 26 ° 52′ 17.4″, longitude 43 ° 36′ 18″. Porosity was measured and permeability was derived from the measured capillary pressure data. The polarization density can be scaled as

$$\mathbf{Sw} = \left[\frac{\mathbf{PD}^{\frac{1}{2}}}{\mathbf{PD}_{\mathbf{max}}^{\frac{1}{2}}}\right]^{[\mathbf{3-Df}]} \qquad 1$$

Where Sw the water saturation, PD the polarization density in coulomb / square meter, PD max the maximum polarization density in coulomb / square meter, and Df the fractal dimension. Equation 1 can be proofed from

$$\mathbf{Q} = \left[\frac{\mathbf{3.14*r^4*\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 2$$

Where Q the flow rate in cubic meter / second, r the pore radius in meter, Δp the differential pressure in pascal, μ the fluid viscosity in pascal * second, L the capillary length in meter.

The flow rate can be scaled as

$$\mathbf{Q = V * A} \qquad 3$$

Where Q the flow rate in cubic meter / second, V the velocity in meter /second, and A the area in square meter. Insert equation 3 into equation 2

$$\mathbf{V * A} = \left[\frac{\mathbf{3.14*r^4*\,\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 4$$

The area can be scaled as

$$\mathbf{A} = \left[\frac{\mathbf{q}}{\mathbf{PD}}\right] \qquad 5$$

Where A the area in square meter, q the electric charge in coulomb, and PD the polarization density in coulomb /square meter. Insert equation 5 into equation 4

$$\left[\frac{\mathbf{V*q}}{\mathbf{PD}}\right] = \left[\frac{\mathbf{3.14*r^4*\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 6$$

The velocity V can be scaled as

$$\mathbf{V = CEK * E} \qquad 7$$

Where V the velocity of flow in meter /second, CEK the electro kinetic coefficient in ampere / pascal *meter, E the electric field in volt/meter. Insert equation 7 into equation 6

$$\left[\frac{\mathbf{CEK*E*q}}{\mathbf{PD}}\right] = \left[\frac{\mathbf{3.14*r^4*\quad\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 8$$

The electro kinetic coefficient CEK can be scaled as

$$\mathbf{CEK = CS * \sigma} \qquad 9$$

Where CEK the electro kinetic coefficient in ampere / pascal * meter, CS the streaming potential in volt / pascal, and σ the fluid conductivity in Siemens /meter. Insert equation 9 into equation 8

$$\left[\frac{\mathbf{CS*\sigma*E*q}}{\mathbf{PD}}\right] = \left[\frac{\mathbf{3.14*r^4*\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 10$$

The streaming potential coefficient CS can be scaled as

$$\mathbf{CS} = \left[\frac{\mathbf{reff^2*CE}}{\mathbf{8*\sigma*\mu}}\right] \qquad 11$$

Where CS the streaming potential coefficient in volt / pascal, reff the effective pore radius in meter, CE the electro osmosis coefficient in pascal / volt, σ the electric conductivity in Siemens / meter, and μ the viscosity in pascal* second. Insert equation 11 into equation 10

$$\left[\frac{\mathbf{reff^2*CE*\sigma*E*q}}{\mathbf{8*\sigma*\mu*PD}}\right] = \left[\frac{\mathbf{3.14*r^4*\Delta p}}{\mathbf{8*\mu*L}}\right] \qquad 12$$

Equation 12 after effective pore radius arrangement will become

$$\mathbf{reff^2} = \left[\frac{\mathbf{8*\sigma*\mu*PD*3.14*r^4*\Delta p}}{\mathbf{CE*\sigma*E*q*8*\mu*L}}\right] \qquad 13$$

Equation 13 after simplification will become

$$\mathbf{reff^2} = \left[\frac{\mathbf{PD*3.14*r^4*\Delta p}}{\mathbf{CE*E*q*L}}\right] \qquad 14$$

The maximum effective pore radius reff max can be scaled as

$$\mathbf{reff^2}_{\mathbf{max}} = \left[\frac{\mathbf{PD_{max}*3.14*\ r^4*\Delta p}}{\mathbf{CE*E*q*L}}\right] \quad 15$$

Divide equation 14 by equation 15

$$\left[\frac{\mathbf{reff^2}}{\mathbf{reff^2}_{\mathbf{max}}}\right] = \left[\frac{\left[\frac{\mathbf{PD*3.14*r^4*\Delta p}}{\mathbf{CE*E*q*L}}\right]}{\left[\frac{\mathbf{PD_{max}*3.14*r^4*\Delta p}}{\mathbf{CE*E*q*L}}\right]}\right] \quad 16$$

Equation 16 after simplification will become

$$\left[\frac{\mathbf{reff^2}}{\mathbf{reff^2}_{\mathbf{max}}}\right] = \left[\frac{\mathbf{PD}}{\mathbf{PD_{max}}}\right] \quad 17$$

Take the square root of equation 17

$$\sqrt{\left[\frac{\mathbf{reff^2}}{\mathbf{reff^2}_{\mathbf{max}}}\right]} = \sqrt{\left[\frac{\mathbf{PD}}{\mathbf{PD_{max}}}\right]} \quad 18$$

Equation 18 after simplification will become

$$\left[\frac{\mathbf{reff}}{\mathbf{reff_{max}}}\right] = \left[\frac{\mathbf{PD^{\frac{1}{2}}}}{\mathbf{PD^{\frac{1}{2}}_{max}}}\right] \quad 19$$

Take the logarithm of equation 19

$$\mathbf{log}\left[\frac{\mathbf{reff}}{\mathbf{reff_{max}}}\right] = \mathbf{log}\left[\frac{\mathbf{PD^{\frac{1}{2}}}}{\mathbf{PD^{\frac{1}{2}}_{max}}}\right] \quad 20$$

$$\mathbf{But;\ log}\left[\frac{\mathbf{reff}}{\mathbf{reff_{max}}}\right] = \left[\frac{\mathbf{log\,SW}}{\mathbf{3-Df}}\right] \quad 21$$

Insert equation 21 into equation 20

$$\left[\frac{\mathbf{log\,SW}}{\mathbf{3-Df}}\right] = \mathbf{log}\left[\frac{\mathbf{PD^{\frac{1}{2}}}}{\mathbf{PD^{\frac{1}{2}}_{max}}}\right] \quad 22$$

Equation 22 after log removal will become

$$\mathbf{SW} = \left[\frac{\mathbf{PD^{\frac{1}{2}}}}{\mathbf{PD^{\frac{1}{2}}_{max}}}\right]^{[\mathbf{3-Df}]} \quad 23$$

Equation 23 the proof of equation 1 which relates the water saturation, polarization density, maximum polarization density, and the fractal dimension. The capillary pressure can be scaled as

$$\mathbf{log\,Sw = (Df - 3) * log\,Pc + constant} \quad 24$$

Where Sw the water saturation, Pc the capillary pressure and Df the fractal dimension

RESULT AND DISCUSSION

Based on field observation the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units as designated in Figure 1. These units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their developed results of the polarization density fractal dimension and capillary pressure fractal dimension are shown in Table 1. Based on the achieved results it was found that the polarization density fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 assigned to sample SJ13 from the Upper Shajara Reservoir as confirmed in Table 1. Whereas the minimum value 2.4379 of the fractal dimension was recounted from sample SJ3 from the Lower Shajara reservoir as displayed in Table1.

The polarization density fractal dimension and capillary pressure fractal dimension were witnessed to increase with increasing permeability as proofed in Table1 owing to the possibility of having interconnected channels. The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which considered as SJ1, SJ2, SJ3 and SJ4 as confirmed in Table 1 were carefully chosen for capillary pressure measurement. Their positive slopes of the first procedure and negative slopes of the second procedure are delineated in (Figure 2, Figure3, Figure 4, Figure 5 and Table 1).

Their polarization density fractal dimension and capillary pressure fractal dimension values are proofed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was reported from 1955 md to 56 md which reflects decrease in polarization density fractal dimension and capillary pressure fractal dimension from 2.7748 to 2.4379 as specified in Table1. Again, an increase in grain size and permeability was recorded from sample SJ4 whose polarization density fractal dimension and capillary pressure fractal dimension was found to be 2.6843 as described in Table 1. In contrast, the Middle Shajara reservoir which is separated from the Lower Shajara reservoir by an unconformity surface as shown in Figure 1. It was designated by four samples (Fig.1), three of which named as SJ7, SJ8, and SJ9 as illustrated in Table 1 were picked for capillary pressure measurement. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in (Figure 6: Figure 8 and Table 1).

Table1. *Petro physical model showing the three Shajara Reservoir Units with their corresponding values of polarization density fractal dimension and capillary pressure fractal dimension*

Formation	Reservoir	Sample	Porosity %	k (md)	Positive slope of the first procedure Slope=3-Df	Negative slope of the second procedure Slope=Df-3	Polarization density fractal dimension	Capillary pressure fractal dimension
Permo-Carboniferous ShajaraFormation	Upper Shajara Reservoir	SJ13	25	973	0.2128	-0.2128	2.7872	2.7872
		SJ12	28	1440	0.2141	-0.2141	2.7859	2.7859
		SJ11	36	1197	0.2414	-0.2414	2.7586	2.7586
	Middle Shajara Reservoir	SJ9	31	1394	0.2214	-0.2214	2.7786	2.7786
		SJ8	32	1344	0.2248	-0.2248	2.7752	2.7752
		SJ7	35	1472	0.2317	-0.2317	2.7683	2.7683
	Lower Shajara Reservoir	SJ4	30	176	0.3157	-0.3157	2.6843	2.6843
		SJ3	34	56	0.5621	-0.5621	2.4379	2.4379
		SJ2	35	1955	0.2252	-0.2252	2.7748	2.7748
		SJ1	29	1680	0.2141	-0.2141	2.7859	2.7859

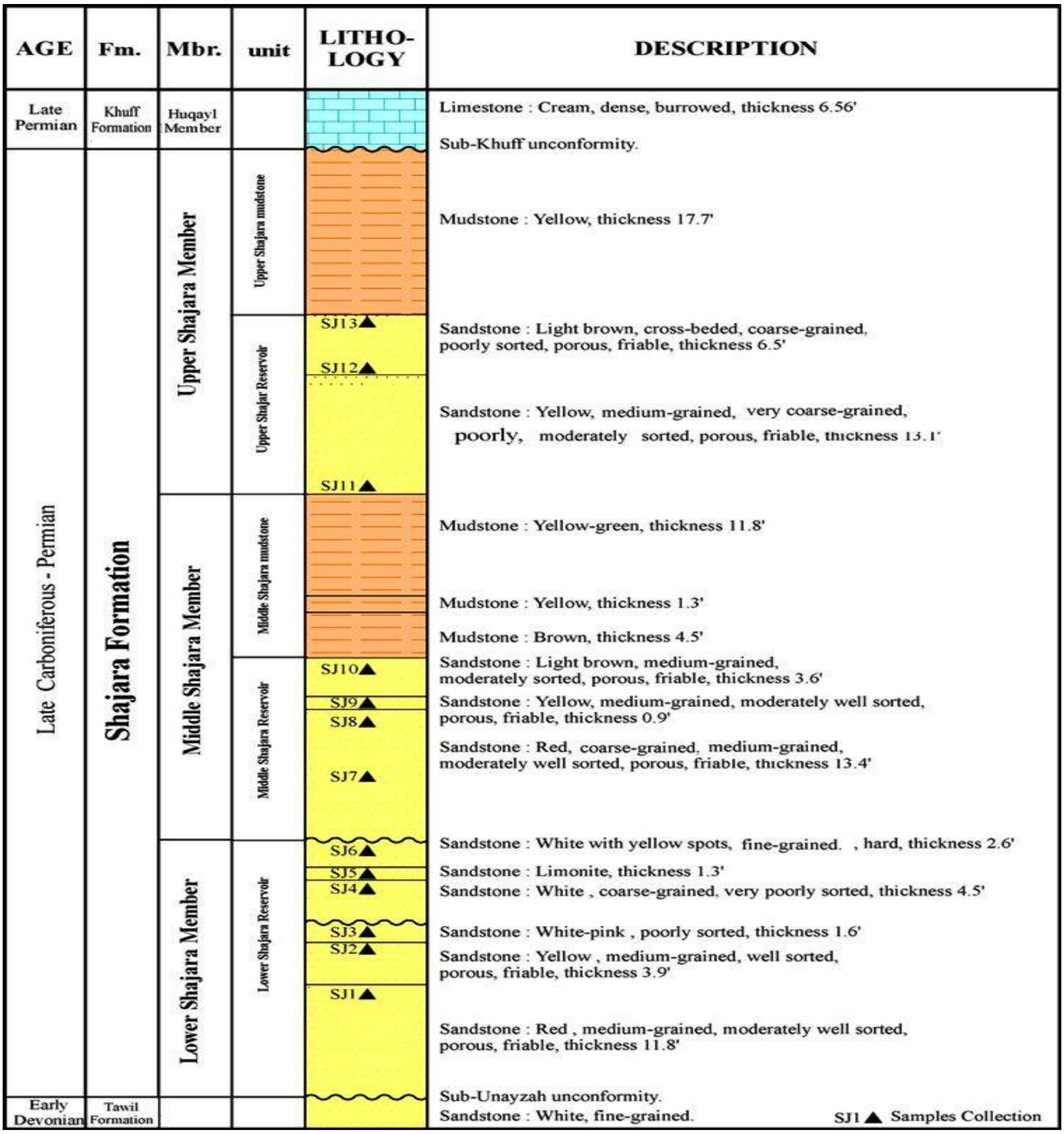

Figure1. *Surface type section of the Shajara reservoirs of the Permo-Carboniferous Shajara Formation at latitude 26° 52' 17.4". longitude 43° 36' 18".*

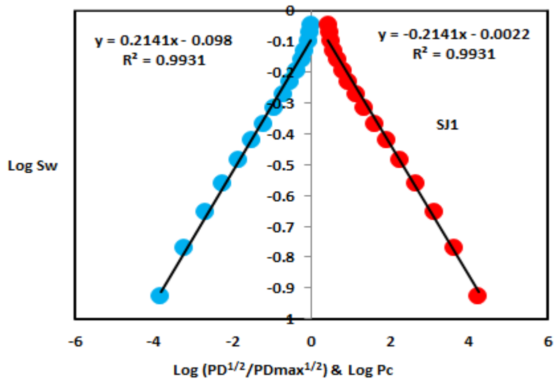

Figure2. *Log ($PD^{1/2}$ /$PD_{max}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ1.*

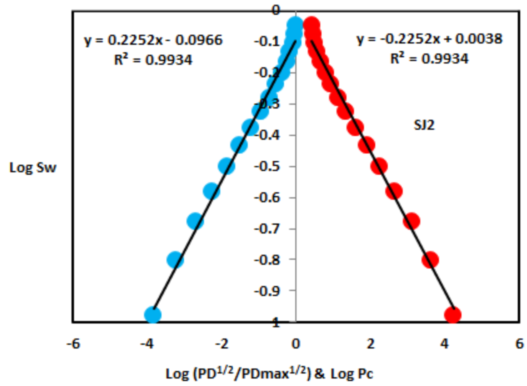

Figure3. *Log ($PD^{1/2}$ /$PD_{max}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ2.*

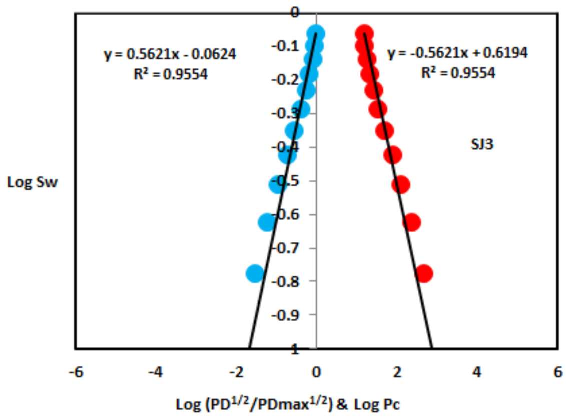

Figure4. *Log ($PD^{1/2}$ /$PD_{max}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ3.*

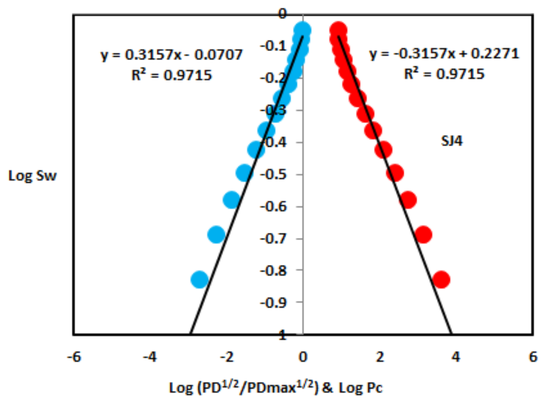

Figure5. *Log ($PD^{1/2}$ /$PD_{max}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ4*

Their polarization density fractal dimensions and capillary pressure fractal dimensions show similarities as defined in Table 1.Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as elucidated increase with increasing grain size and permeability in Table1.

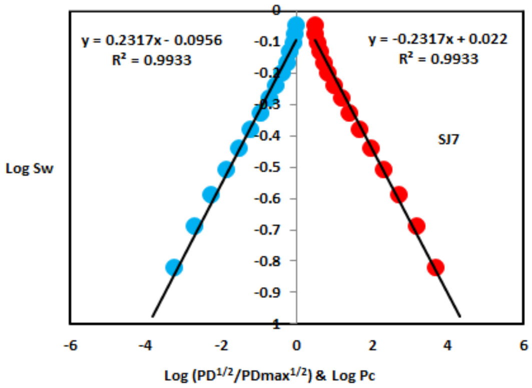

Figure6. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ7.*

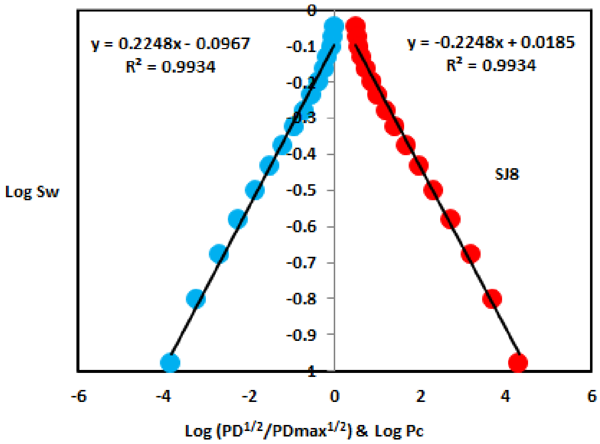

Figure7. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ8.*

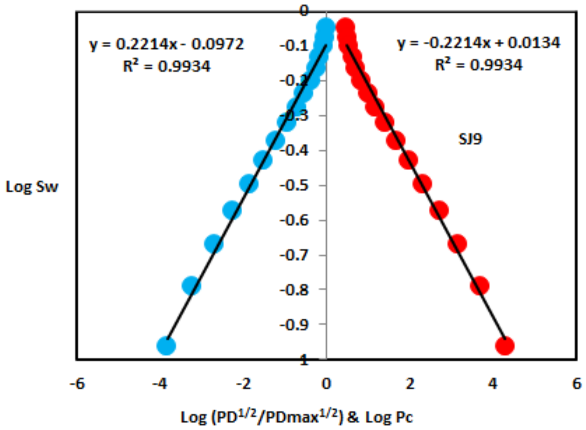

Figure8. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ9*

On the other hand, the Upper Shajara reservoir is separated from the Middle Shajara reservoir by yellow green mudstone as revealed in Figure 1. It is defined by three samples so called SJ11, SJ12, and SJ13 as explained in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are exhibited in (Figure 9, Figure10, Figure 11 and Table 1). Moreover, their polarization density fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in Table 1.

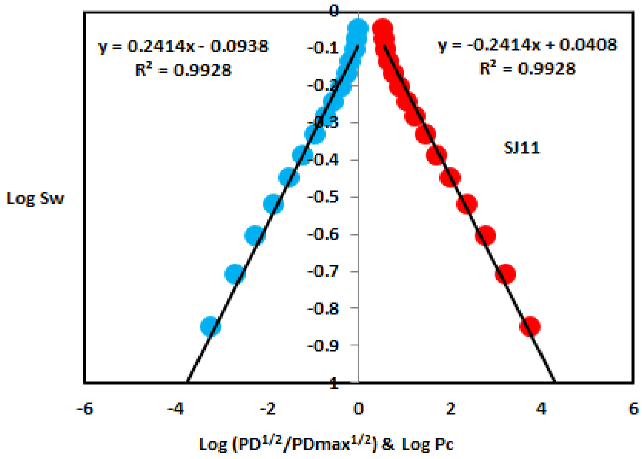

Figure9. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ11*

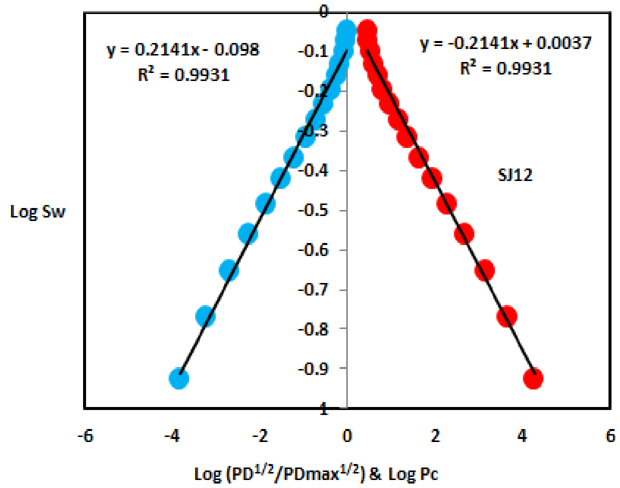

Figure10. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ12.*

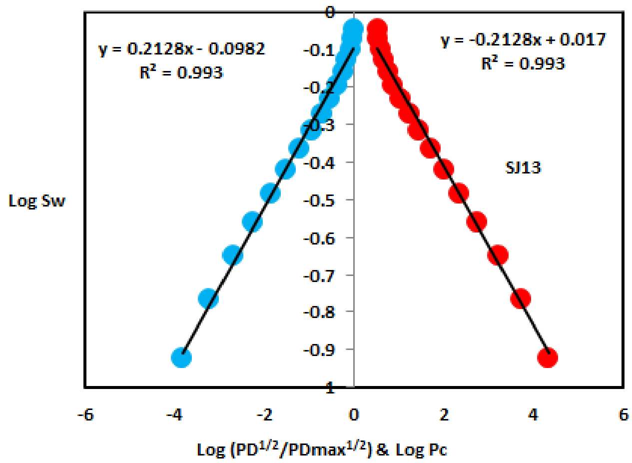

Figure11. *Log ($PD^{1/2}$ /$PD_{max}{}^{1/2}$) versus log Sw and log Pc versus Sw for sample SJ13.*

Overall a plot of polarization density fractal dimension versus capillary pressure fractal dimension as shown in Figure 12 reveals three permeable zones of varying Petro physical properties. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment. This heterogeneity was also confirmed by plotting positive slopes of the first procedure versus negative slopes of the second procedure as proofed in Figure 13. Increase with increasing grain size and permeability.

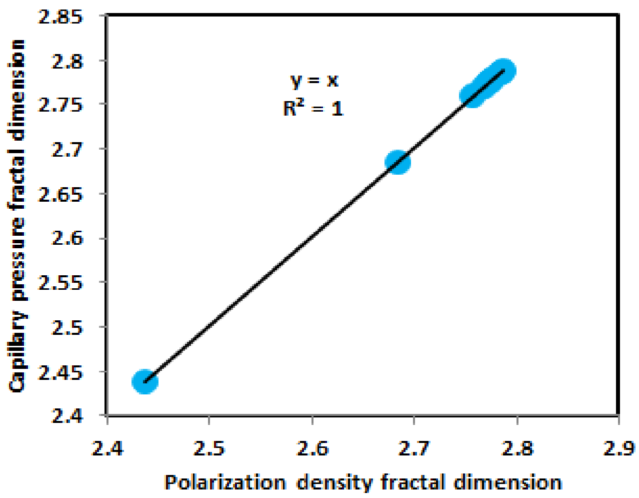

Figure12. *Polarization density fractal dimension versus capillary pressure fractal dimension.*

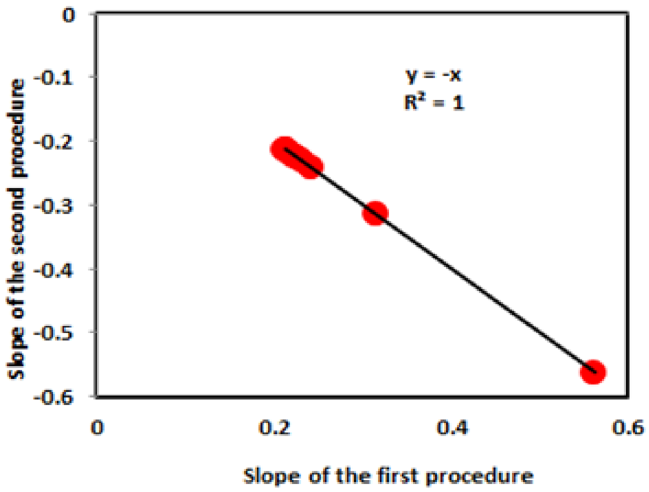

Figure13. *Positive slope of the first procedure versus negative slope of the second procedure.*

CONCLUSION

The sandstones of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units based on polarization density fractal dimension. The Units from bottom to top are: Lower Shajara polarization density Fractal dimension Unit, Middle Shajara polarization density Fractal Dimension Unit, and Upper Shajara polarization density Fractal Dimension Unit. These units were also proved by capillary pressure fractal dimension. The fractal dimension was found to increase with increasing grain size and permeability.

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