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Different and Dissonant Values in Measuring Dimensions in Ancient Egypt "A Comparative Study with Contemporary Measurements"

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ABSTRACT

This paper focuses on How there are different and dissonant values in measuring dimensions in ancient Egypt? The ancient Egyptians relied on a natural method to measure dimensions like the arm that was used as a measure of length, approximately equal to the length of a forearm. Traditionally, it was the length from the bent elbow to the tips of the fingers. Typically, almost 18 inches or 44 cm, however there was a long cubit of about 21 inches or 52 cm. The second natural method was the width of the palm of the hand. As well as, the human fingers used as digits for measuring width, where the four digits equal the sign of one palm and seven palms equals one cubit. In present-day trigonometry, cotangent requires same units for both horizontal run and vertical rise, however ancient sources like Rhind Papyrus uses palms for the run and cubits for the rise, resulting in these different, yet characteristic mathematics. In ancient Egypt there were seven palms in a cubit, in addition to the seqed that was seven times the cotangent. There are some questions are as follows: Is it true that the Egyptian seqed is the ratio of the run to the rise of a slope of a cotangent? How to measure the inclination in ancient Egypt accurately? Is the value of the seqed or the angle θ correct by applying the rules of modern trigonometry? The question arises as to whether the problems of the seqed are actually mirrored in the ancient Egyptian architecture methods and construction techniques? How are there different and dissonant values in measuring dimensions in ancient Egypt?

Keywords: *Mathematics, Measuring, Dimensions, Inclination, Dissonant Values*

INTRODUCTION

Science is a matter of asking for information by specifying the difference between fact and opinion. Fact in a scientific context is a generally accepted reality, but still open to scientific inquiry, as opposed to an absolute truth, which is not a part of science. The hypotheses and theories are generally based on objective inferences, unlike opinions, which are generally based on subjective influences. Therefore and hence we can say 'the facts versus the opinions'. In determining the factual length of an ancient Egyptian cubit or the true value of a seqed and Inclination, as discussed in the Rhind and Moscow Mathematical Papyruses, one will notice confusing differences. Varying opinions have been advanced, e.g., by Carter & Gardiner, Iversen, Hayes, Budge, Noblecourt, Lorenzen, Müller, Gardiner, Stricker, Gay, Legon, Gay & Shute, Naguib, Roik, Shaffer, and the Encyclopaedia Britannica. Possibly, though, also ancient Egyptian sources might have offered different values. The question now is how to measure a seqed and inclination in ancient Egypt accurately? Is the value of the seqed or the angle θ correct by applying the rules of modern trigonometry?

The question arises as to whether the problems of the seqed are actually mirrored in the ancient Egyptian architecture methods and construction techniques? How is it possible that one can actually measure the seqed in some pyramids whilst in others a different or adjusted method was used to determine the pyramid's sloping angle? Although scientists differ on how to measure and determine the true value of a seqed, we can assume that the ancient Egyptians depended on a natural method of measuring dimensions such as the cubit, which was used as a measure of length, approximately equal to the length of a forearm. Traditionally, this was measured from the bent elbow to the tips of the fingers, measuring approximately ≈44 cm (≈18 inches), and a long cubit of about≈52 cm (≈21 inches). Noteworthy that these numbers given are not in accordance with fact in comparison with modern units, where 18"= 45.72 cm, and 21"= 53.34 cm. The second method was using the span, i.e. the width of the palm of the hand, with the human fingers used as digits of measuring the width. Four digits equalled the sign of one palm and seven palms equalled one cubit. In present-day trigonometry, the cotangent requires the same units for both the horizontal run and vertical rise. Ancient sources, however, such as the Rhind Papyrus dating back to the First Intermediate Period, 16 century B.C., which uses palms for the run and cubits for the rise, resulting in characteristically different, mathematics. In ancient Egyptian mathematics there were seven palms in a cubit, in addition to the seqed, which was seven times the cotangent. The Egyptian seqed is the ratio of the run to the rise of a slope of a cotangent.

The Rhind Papyrus that is an ancient Egyptian mathematical document mentions the seqed repeatedly (e.g. 56, 57, 58, 59, 59 b and 60), in connection with many problems or issues. The ancient Egyptians knew about the concept of a slope which is equal in value to the cotangent of a tangent angle or along a tangent point. The tangent is a line, curve, or surface meeting another line, curve or surface at a common point and sharing a common tangent line or tangent plane in that point. The abbreviation 'tan' is the trigonometric function of an acute angle in a right triangle that is the ratio of the tan θ, which is equal in value to the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle. Ancient Egyptians used the tangent angle by identifying it as a seqed, which is the ratio of two lengths of the sides of a right-angled triangle. The ancient Egyptians used the equivalent of similar triangles to measure dimensions and lengths of slopes defining the seqed as the ratio of the run to the rise, which is the reciprocal of the recent definition of the slope. For example, the seqed of a pyramid is considered as the number of palms in the horizontal corresponding to a rise of one cubit or seven palms. Thus, if the seqed is 51/4 and the base is 140 cubits, the height becomes 931/3 cubits that is mentioned in the Rhind Papyrus (issue No. 57). The ancient Egyptian calculations indicate that the seqed is equal in value to one Royal cubit, which equals seven palms or twenty-eight fingers. Mathematically, the seqed can be expressed by the following equation; Seqed = 1 Royal cubit = 7 Palms = 28 Digits/Fingers. The ancient Egyptians have not measure angles by a direct way, but used the theory of a slope which is equal to the tangent of an angle; their angle concerning measure was the seqed, which is defined as the rate of two lengths of the sides of a right triangle taken in a certain method, where the seqed is the length from a human elbow to the end of the middle finger, the approximate width of the human hand is equal a palm, and the estimated width of a finger is equal in value to a digit. The ratio of the two shorter sides of the triangle is measured in cubits for the vertical side (H) and in palms along of the horizontal side (W), wherefore the seqed $= \frac{\text{Width by Palms}}{\text{Height by Cubits}} = 7\tan(\theta)$. Here θ is the angle between the vertical and the plummet line. The 7 tan (θ) is due to the fact that one cubit equal in value to seven palms. For example, the secant of the angle θ equals in value to the ratio of the length of the face of the Great Pyramid of Giza or the slope height, inclined at an angle θ to the surface of the earth, to half-length of the side of the square base.The objectives of this study will be detected through discussion, investigation, analysis that can be detected through detailed interrogation of the objects, elements and structure of the content of this study, particularly as a basis for explaining the content and significance through the discussion and interpretation. All of this will be clear through the methodology, structure and content of this study.

The Methodology and Structure of the Study

A Survey of Different and Dissonant Values in Measuring Dimensions in Ancient Egypt

There are some differences which have caused confusion when we want to measure the length of a cubit or to determine the accurate value of the seqed and inclination especially or dimensions in ancient Egypt generally. The Egyptian seqed is the ratio of the run to the rise of a slope of a cotangent, broadly based on the ancient Egyptian measurements of the Royal cubit, the palm and the digit **[1]**. It should be mentioned that the confusion and the difficulties explaining the seqed and inclination problems arise in part from the technical words employed. EISENLOHR, the first editor of the Rhind Papyrus, mentions the different interpretations by scientists as early as 1877. He was well aware of the difficulties related to the issues of a seqed and inclination.

EISENLOHR concluded that the term seqed referred to the ratio of two lines. Also, he concluded that the seqed should refer to either the ratio of the half-base to the apothem or to the ratio of the half-diagonal to the sharp edge formed by the meeting of two flat or curved surfaces. In contemporary terminology, these ratios correspond to cos (α) and cos (β) respectively **[2, 3]**. Moreover, PETRIE (1877) argued that the cubit derived from the original value of the typical average, which may be slightly increased by continual copying over the ages **[4].**

CARTER & GARDINER (1917) emphasized that a cubit measured 523 millimetres; this is according to information about the tomb of Ramesses IV mentioned in the Turin plan of a royal tomb **[5]**. Later, it was studied again by WEEKS within the frame of the Berkeley Theban Mapping Project/ BTMP. WEEKS compared the measurements of the papyrus and those taken by the BTMP, and then used CARTER's suggested value of the cubit of 1 Cubit = 0.5231 meters **[6]**. IVERSEN (1955) believed that Egyptian metrology is based on measures of 4 small cubits and that two canons were used successively. The unit of the first canon was the fist and, when squared to form a grid, it automatically divided the height of the standing male figure into 18 squares. According to the second canon the height of the figure was divided into 21 squares. He assumed the reason for the change to be found in metrology: about the time of the 26th Dynasty the small cubit was replaced for all practical purposes by the royal cubit **[7]**. HAYES (1957) referred to this fact by studying a 22 cm high diorite statuette of Sennemut (now kept in the Metropolitan Museum of Art, accession No. 48.149.7) which is a reproduction at a scale of 1:7 (i.e. one palm to the cubit) of Cairo CG 579 showing Sennemut presenting a Sistrum to Mut. The long autobiographical text which occupies the back pilaster of Cairo CG 579 finds no place at all on its small replica **[8]**. BUDGE (1960) clarified that a cubit is 0.525 meters **[9, 1]**. NOBLECOURT (1965) mentioned that in King Tutankhamun's tomb, four models of the famous Egyptian unit of length were found and that they measured one foot and seven and a half inches **[10, 1]**. LORENZEN (1970) indicated that there was a system labelled the single-inch system consisting of a 19 squares grid each square being 5 inches. The top edge of a square inscribed in the circle is approximately mid-way between 5 x 19 fists and 4 x 24 fists. This is why the first canon pattern is 19 squares high. Noteworthy that the basic measure or the prevalent module during the 18th-19th dynasties was the so-called 'Scale A1' or the double-inch system which equals 5-inch per fist(the hand-unit of scale A). The fathom, the royal cubit, and the foot remained unchanged as 3/4, ¼ and 1/8 of the diameter but became subdivided into 16 double-fists = 32 half double-fists = 128 half double-inches (written 128"). This leads to the equation: one fathom =3 royal cubits = 4 pechys = the length between the roots of the thumbs when the arms are outstretched **[11]**. MÜLLER (1973) discussed the royal and the smaller (or normal) cubit, and explained how the proportions of the human body became 'translated' into a canon. He then argues that the Old Kingdom system of axially crossing lines became in the Middle Kingdom replaced by a grid adjusting not only the width of the figures but also the lay-out of entire walls. The system remained unaltered during the New Kingdom and afterwards, but in the Saite Period one passed from the royal to the smaller cubit meaning that the height of the standing figure was henceforth 28 instead of 24 handbreadths. This resulted in an 'unnatural' length of the feet and slimness of the bodies **[12]**. GARDINER (1973) noted that the value of a cubit totalled 20.6 inches or 523 millimetres **[13, 1]**. STRICKER (1976) believed that the Egyptian royal cubit measured seven palms, and the small cubit six palms. He states, for instance, that the length of one cubit of the newly-born children in the Pap. Westcar alludes to innocence, the cubit being the latter's measure also in other instances **[14]**. GILLINGS (1982) suggested that the seqed was the ancient Egyptian unit of measure for the inclination of the triangular faces of a typical pyramid. The measurement method was based on the Royal cubit of seven palms subdivided into four digits. The inclination of slopes was thus expressed as a value of horizontal palms and digits for every Royal cubit rise. This inclination may be a type of the contemporary measure belonging to the so- called Gradient which is in connection with the contemporary concept of cotangent and tangent **[15]**. Gay (1985) noted against IVERSEN's explanation for the change in the canon of proportions during the 26th Dynasty that a change in the grid system preceded any changes in the metrical system. In order to conform to the 6 palms of a small cubit the forearm was divided into 6 squares instead of 5. This allowed to generate a new grid system for standing figures from the old one using increments of one fifths of a square. These subdivisions, although realized in a few models from the Middle Kingdom onwards, were rarely used with such accuracy, however; the new grid system, therefore, caused only minimal changes in the principal ratios **[16]**. GAY & SHUTE (1985) concluded that in ancient Egyptian buildings with inclined walls (such as pyramids and pylons) the inclination was contrived to give a horizontal displacement measured in palms, half-palms or quarter-palms for a drop of one royal cubit of seven palms. The inclinations of the edges between adjacent walls were not relevant to the design. Evidence is presented suggesting that in two-dimensional art the obliquity of slanting lines may have been determined by a similar system, but with a drop of six units instead of seven. After the introduction of the squared grid–obliquity was probably controlled by reference to grid intersections. So, there is no valid cause for assuming that the irrational values□ □/ pi and □ / phi were involved, except coincidentally in the inclination angle of the pyramids or in the slope of slanting lines **[17]**. As is well known that ratio of the circumference to the diameter, '□/pi'(≈3.1416) and the proportion of 'golden section', □ / phi (≈1.6180). LEGON (1988) believed that the precision with which the Great Pyramid of the King Khufu had been built suggests that the builders had wanted to express particular dimensions as accurately as possible. Only limited or partially accurate estimations of the use of the Royal cubit in this pyramid have hitherto been put forward. A number of correlations between the various dimensions have thus gone unnoticed, he assumed, with the result that the intentions of the architect have not been properly understood. Far from being the outcome of a series of changes of plan the arrangement of the passages in the Great Pyramid reveals a logical and highly-integrated design according to his argument **[18]**; whilst GAY & SHUTE (1990) explained and defended their view on the 14 to 11 proportion. They maintain that the size and the shape of a pyramid were predetermined by the lengths of the sides of its square base and by the slope of its triangular faces. The unit of measure was usually the seqed relating to lateral displacement in palms for a vertical drop of seven palms or one royal cubit. The seqed of the Great Pyramid, e.g., is believed to be 51/2, which converts into an inclination of 51(51'). The other interpretation of pyramidal form lays particular emphasis on the height being more hypothetical since the height, unlike the slope, cannot be checked directly at all building stages. The approximation of the values '□/ pi and □ / phi' are discussed and this last value or number derived from the golden ratio. The square root of 'phi' relates to the height of the Great Pyramid as a result of the use of a seqed of 51/2. But it does not follow from this that the ancient Egyptians knew of this mathematical ratio or that they had any concept of phi and its square root **[19]**. NAGUIB (1991) believed that the rod (nbi) was a measure of length where longer linear units were needed believing it to be a full arm of 70 cm length. He sees a relation between royal cubit and rod: 1 rod = 1 1/3 royal cubit and 1 royal cubit = 3/4 rod [**20**]. By way of example he cites the rod and its divisions having been used for the principal architectural features of the tomb of Hem-Min (M43) at the cemetery of El-Hawawish at Akhmim (end of the 5th/-beginning of the 6th Dynasty) **[20-23]**. ARNOLD (1991) considered the possibility that the Egyptian cubit was longer than a typical forearm. It seems to have consisted of 7 palms of 4 digits, which equals 28 units of altogether 52.3 to 52.4 cm in length **[24]**. The earliest certified gauge is the royal cubit of about 523 to 525 mm (= 20.6 to 20.64 inches). In ancient Egyptian architecture the royal cubit is known from the Old Kingdom onwards **[24-26]**; whilst ROIK (1993) considered the royal cubit as the basic measure of length in ancient Egypt, and set forth her views about the existence of a yet undetected ancient Egyptian measuring system basing her reasoning on measurements found in the tomb of Tawosre. She explored the evidence for this system, which she sees based on the existence of the nbi (Greek naubion?), a measure of length provisionally estimated to be about 65 cm. Also, she concluded that the nbi system was used as basis of the grid furnishing examples of measurements of the seated and standing human body **[27]**. LEGON (1996) examined the evidence for the use of the small and royal cubits in Egyptian monuments, concluding that the Egyptian artists' canon of proportions was based on the royal cubit and not upon the small cubit as hitherto claimed. The 'canonical height' of the standing figure is thus found to have been three royal cubits. The length of the cubit is shown to have been divided in practice into simple fractions, as well as into the more customary units of palms and fingers **[28]**. According to SHAFFER (1996) the cubit is a measure of length from the elbow to the end of the middle finger; (18 inches or 45.72 centimetres) **[29, 1]**. The ENCYCLOPAEDIA BRITANNICA/MERRIAM-WEBSTER(2016) describes the cubit as an ancient unit of length based on the length of the forearm from the elbow to the tip of the middle finger and usually equal to about 18 inches or 46 centimetres **[30, 1]**. The value of the seqed and inclination especially or dimensions generally in ancient Egyptian architecture varies therefore. The slopes of the faces of the Great Pyramid of Giza measure a seqed of 5 1/2, or 5 palms and 2 digits, which amounts to a slope of 51.84° **[31]**. EDWARDS (1979) considered this to have been the 'normal' or most typical slope choice for pyramids **[32]**. A lot of the smaller pyramids in Egypt show other slope angles. Although looking like the Great Pyramid of Giza, the pyramid at Meidum is thought to have a slope of nearly 51.842° or 51° 50' 35" amounting to a seqed of 5 1⁄2 palms **[33]**. The similarity of the slope at Meidum and at Giza is believed to be deliberate wishing to make certain that the circuit of the base of the pyramids was exactly comparable to the length of a circle, this if the pyramid's height were used as a radius **[34]**. Noteworthy that these relations of areas and of circular ratio are so systematic those were included in the builder's design **[35]**.

Semantics and Linguistic Review of Different and Dissonant Values in Measuring Dimensions in Ancient Egyptian Language

The linguistic structure of the seqed and inclination especially or dimensions in ancient Egyptian language generally reflects their alternative forms. In this regard, there are some written forms indicating the meaning of 'slope angle', 'tangent angle' as well as 'to measure the cotangent point'. These written forms were as follows; , **'Sqd'** meaning 'slope of pyramid' **[36-42]**. Mathematically, it means 'degree or extent per cubit in height, altitude or amount, which is half the baseline divided by the Pyramid height, expressed in handbreadths a unit of measure angles'**[41-44]**. It has been noticed that different determinatives or ideograms were attached to the previous word, such as , **[13]**. Gardiner's sign-list classifies (Aa 28) as a representation of the ideogram used in the verb , **'Qd'** that means to 'build'. The instrument represented by the ideogram was used by bricklayers; the sign sometimes stands alone such as in the scene of the funerary buildings of the King Saḥu-Ra **[13, 45-54]**. Another suggestion is a striker used in measuring corn **[55-56]**. Another form of the first ideogram is , which was used in the Old Kingdom and continued until the New Kingdom **[13, 33, 45, 49, 57-60]**. In Hieratic , is usually transcribed as **[61]**. Furthermore, there were other related words, previously mentioned such as; **'Sqd/Sqdy'** meaning 'line of measure', , , **'Qd'** meaning 'to extend, length, ratio, dimension', **'Qddw'** meaning 'characters of people, who measure the extent or the length', , , **'Qdwt',** which means 'sketch', or the action of indicating the exact position or location of a border or a borderline **[40]**. Also **'Sqd'** meaning 'length, ratio, dimension' **[39-40, 46]**.This is beside the written form of the word **'Sqdw'** meaning 'builders', perhaps used for 'those who commissioned the work of measuring slopes'**[37, 39]**. In addition **'Qdt'** that means **'Kite'** (= weight of one tenth of a Deben or 9.1 grams' **[36, 40, 62-71]**. In Coptic the variant forms of the writing were **'Kite'** and **'Kit'** **[72-74]**. In the Sahidic Dialect the written forms were **'Kwte'**, **'Kwt'** and **'Kot'[75-77]**. The original meaning 'pot' led via the mythological picture of god Khnum forming man on a potter's wheel to the meaning of **'Qd'** 'personality, character' as can be found in the expression 'the day of the reckoning of the characters/ **hrw Hsb qdwt**' referring to Judgement of the deceased in the Osirian 'Hall of Judgement'**[78-79]**.

Different and Dissonant Values in Measuring Seqed, Inclination and Sloping Span Length in Ancient Egypt

GILLINGS (1982) defined the seqed, inclination and sloping span length of a pyramid as follows: 'The seqed of a right pyramid is the inclination of any one of the four triangular faces to the horizontal plane of its base, and is measured as so many horizontal units per one vertical unit rise. It is thus a measure equivalent to our modern cotangent of the angle of slope. In general, the seqed of a pyramid is a kind of fraction, given as so many palms horizontally for each cubit of vertically, where 7 palm equal one cubit. The Egyptian word 'Seqed' is thus related to our modern word 'Gradient' **[15]**. The method for the measuring of a sloping span length in ancient Egypt becomes clear looking at religious and funeral structures. A very impressive structure is the great pyramid of King Khufu on the Giza plateau **[80-89]**. PETRIE (1883) believed that for an accurate determination of the value of the usual Egyptian cubit the King's Chamber in the Great Pyramid (called 'Khufu's Horizon') was certainly the most significant source as it is the most accurately built best preserved and the most precisely measured **[31]**. NICKEL (2009), MAGLI (2010), BRYN (2010), and WAKEFIELD (2016) agree with what PETRIE mentioned above **[90-93]**. It should be noted that the most important aspect of pyramid construction was the precise mathematical calculation and determination of the sloping span length of a structure, which was kept carefully close to the line of the horizon **[82, 90-93]**(Figs.1-5).

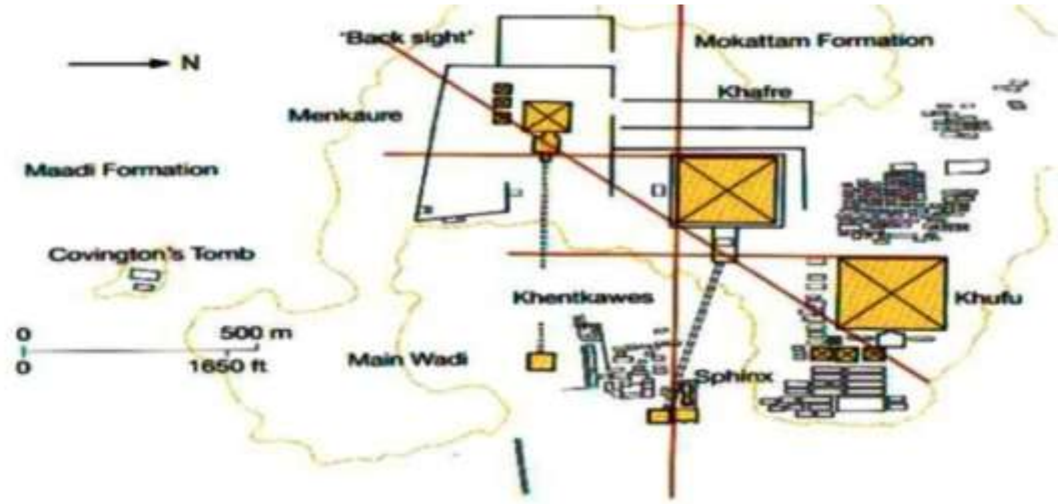

Fig1. *Map of the main Giza pyramids with the Giza axis highlighted (after: Lehner 1999; Magli, 2010)*

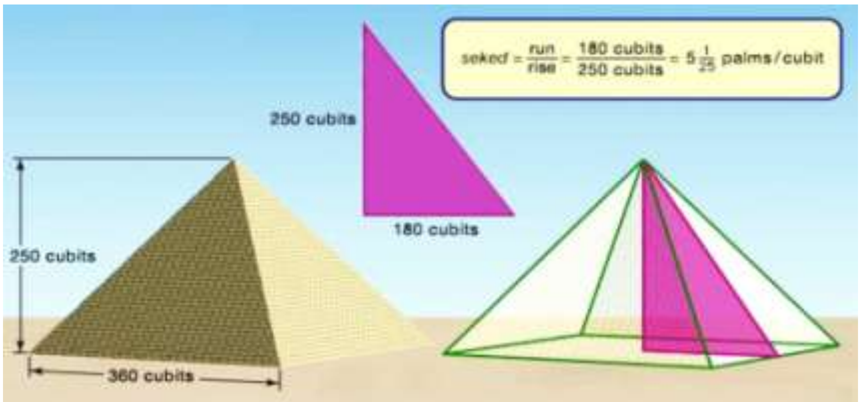

Fig2. *The Seqed of a pyramid is considered to be the number of palms in the horizontal corresponding to a rise of one cubit or seven palms (after: Merriam-Webster's International Encyclopædia Britannica Online Inc., 2016)*

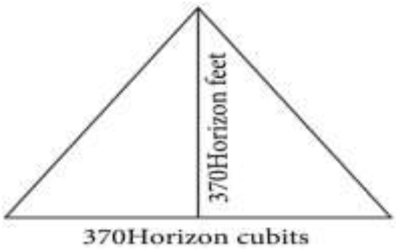

Fig3. *The average width of the base of the Great Pyramid amounts to about 370 'Horizon cubits', the disparity is nearly 1 portion in 6,000 or 0.016%, and therefore the approximation number will be supposed to be the planned or meant width. This pyramid was apparently intended to conform to the proportions of a Pythagorean {3-4-5} triangle, giving a theoretical height of 246 ⅔ cubits or 370 feet (after Wakefield, 2016).*

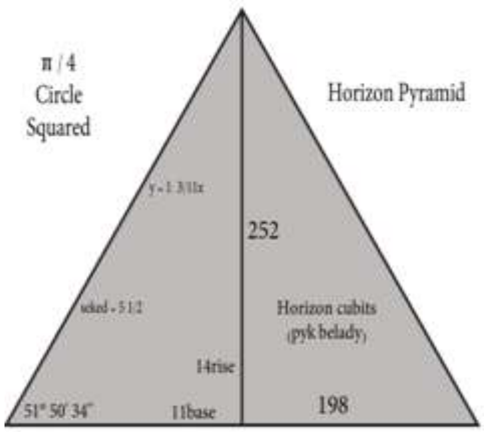

Fig4. *Theoretical Proportions and Theoretical Cross-Section of the 'Horizon' Pyramid (after Wakefield, 2016)*

The Giza necropolis with The-6 square grid of the master plan

a. Khufu's Great Pyramid; **b.** Khafre's Pyramid; **c.** Queen's Pyramids; **d.** Cult Pyramids **e.** Mortuary temples **f.** Valley temples; **g.** Causeways. The-6 square grid; **h.** The Sphinx and temple; **i.** Royal Cemetery; **j.** Mastabas; **k.** The trial passage

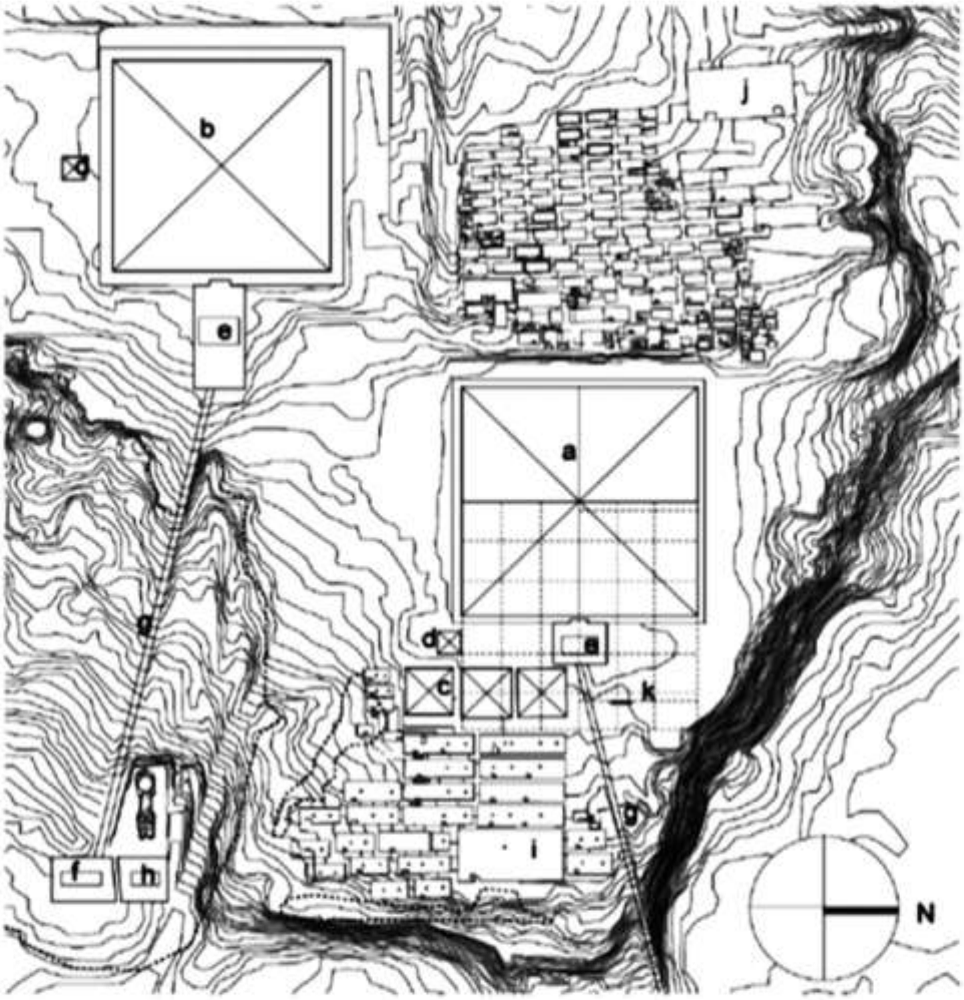

Fig5. *Diagram illustrating the building processes on the Giza plateau (after BRYN, 2010)*

The ancient Egyptians knew about the concept of a slope which is equal in value to the cotangent of a tangent angle or along a tangent point. The tangent is a line, curve, or surface meeting another line, a curve, or surface at a common point and sharing a common tangent line or tangent plane in that point. The abbreviation 'tan' is the trigonometric function of an acute angle in a right triangle that is the ratio of the tan θ, which is equal in value to the length of the side opposite the angle to the length of the side adjacent to the angle **[90-93]** (Fig. 6).

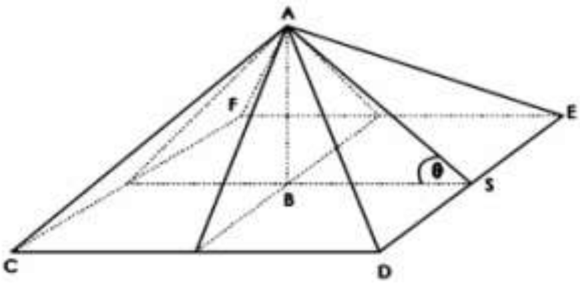

Fig6. *Diagram illustrating the calculated seqed that is the measure of an angle θ. In current trigonometry seqed (or θ) means the cotangent of the angle between the base of the pyramid and one of its faces. Thus, if the tangent of an angle is a/b then the cotangent of that angle is b/a. It is noteworthy that the right triangle (ABS), where (AB) is the height and (BS) is half the length of the base (after Nickel, 2009)*

The preceding diagram explains the concept of the right triangle (ABS), where AB is the height and BS is half the length of the base. The Rhind Papyrus is clearly dealing with this type of triangle because the solution involves calculation with half the value of the base; that is to say $\frac{1}{2}$ of 360 = 180 the reason being that the tangent value in this triangle is $\frac{AB}{BS}$ and the complementary rate or the cotangent angle is $\frac{BS}{AB}$. The two angles are complementary if the total of them is meant to be 90°. It is noteworthy that the two acute triangles of a true triangle always emphasize the values of each other and it should be symmetrical or integral. It means that the tan θ equals cot (90° – θ), where 90° – θ equals ∠BAS, and cot θ equals tan (90° – θ). Additionally, tan θ equals $\frac{AB}{BS}$ and cot θ equals $\frac{BS}{AB}$ in a right triangle. For this reason, there is a mutual benefit or a reciprocity relationship between the tangent rates and cotangent values **[90, 94]** (Fig.7).

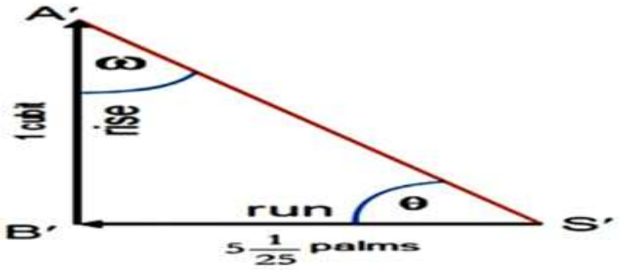

Fig7. *Most modern scientific calculators offer no function for finding the inverse cotangent using the complement of the cotangent, i.e. the tangent instead. We set ω = 90° – θ. Hence, ω = tan-1 (0.72) ⇒ ω = 35.75° (the complement of the angle we are looking for). Since ω = 90° – θ, then θ = 90° – ω. Hence, θ = 90° – 35.75°= 54.25°. This value corresponds closely to the actual inclination angles of the pyramids of Egypt (after NICKEL, 2009)*

Fig8(a). *Problem No. 56 of the Rhind Papyrus, written in Hieratic by the scribe Ahmose (c. 1550 B.C). (after IMHAUSEN, 2016); http://www.britishmuseum.org/system_pages/beta_collection_introduction/beta_collection_object_details/beta_collection_image_gallery.aspx?assetId=366139001&objectId=110036&partId=1#more-views http://www.britishmuseum.org/)*

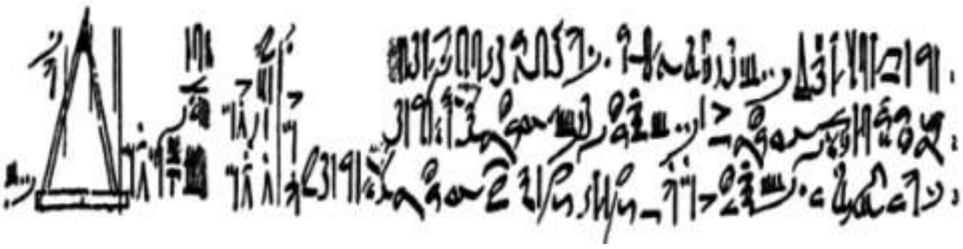

Fig8(b). *Facsimile of problem No. 56 of the Rhind Papyrus (after http://www.britishmuseum.org/system_pages /beta_collection_introduction/beta_collection_object_details.aspx?assetId=766120001&objectId=117389&partId=1 http://www.britishmuseum.org/)*

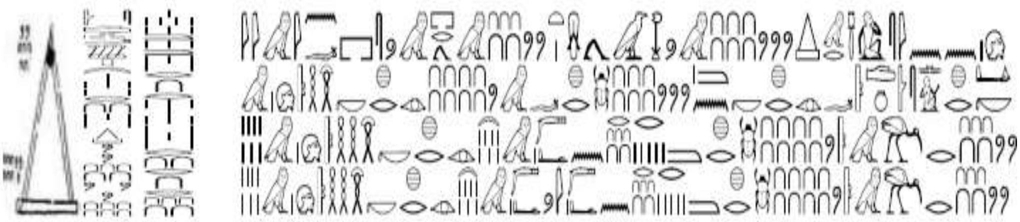

Fig8(c). *Hieroglyphic transcription of problem No. 56*

The text accompanying issue No. 59a reads as follows: 'If the side of a pyramid is 12 cubits and the height are 8 cubits what is its seqed'? The solution will become clear considering the following; Take ½ of the side. The result: 6. Determine the fraction of the height 8 which gives 6? The result: ¾. Then 1 cubit = 7 palms; 1 palm = 4 fingers. Take ¾ of 7 palms. The result: 5 palms and 1 finger; this is the seqed **[2-3, 44, 102]**. There is a noteworthy belief that metrological standards in ancient times matched the same principles that are used in the present; therefore, the value of a seqed would have possessed exactly the same absolute value in whatever the context. It would appear that in monumental architecture units were consistent only within the framework of one particular building project. A certain set of standards would be created for each pyramid, e.g., and ritually dedicated specifically for that pyramid **[31, 90-93]**. The average width of the base of the Great Pyramid is about 370 'Horizon' cubits ('pyk baladi'. Etymologically, the term 'pyk belady/baladi' is related to the Greek 'pekhus and pygon', both also meaning a cubit, with that being the number of cubits to the Greek stade (about 600 feet [180 metres]). Hence 'pyk belady' can be taken to mean 'national cubit [of Egypt]'), therefore the difference amounting to just1 part per 6,000 or 0.016%. For our purpose we assume the approximate figure corresponding to the planned or desired width. Mathematical ratios existing in this pyramid seems to be close to the proportions or values of a Pythagorean (3-4-5) triangle resulting in a theoretical height of $246\,{}^{2}/_{3}$ cubits or 370 feet. In fact, it is well known that the angle θ for the Pythagorean triangle is 53.13°, which is more than 1° greater than the angle for the Khufu's Pyramid. The seqed of a pyramid is calculated by finding **x** in terms of **y** then multiplying the coefficient of the determination of **y** by **7**. Or the other way round: it is **7** times the cotangent of the pyramid's dihedral angle. In order to determine the slope of an angle the latter should be considered as being formed by a horizontal line and a slanted line as illustrated below. Such an angle can be defined by just giving its slope. In the illustration the slope of angle A is the ratio h/b. In geometry this mathematical relation is also referred to as the tangent of the angle A and denoted by Tan (A). The angle A can also be specified by cot (A) =b/h, which is called the cotangent of the angle A and is opposite of the slope **[98,100,106-108]** (Fig.9).

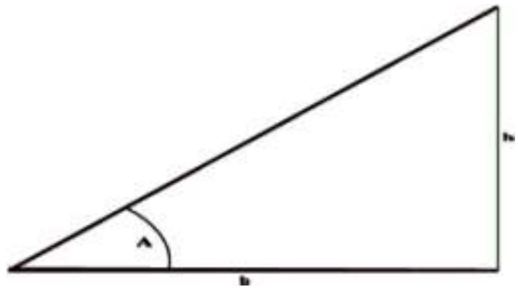

Fig9. *Diagram illustrating angle A, which can also be specified by cot (A) =b/h (cotangent of the angle A), which is opposite of the slope) (after: https://www.math.washington.edu/~greenber/slope.gif)*

The ancient Egyptian mode of calculation shows the seqed to equal the value of one Royal cubit of seven palms and twenty-eight fingers. Mathematically, the seqed can be expressed by the following equation:

$\text{Seqed} = \frac{\text{Width by Palms}}{\text{Height by Cubits}} = 7\tan(\theta)$ whereby 1 Seqed equals 1 Royal cubit = 7 Palms = 28 Digits/Fingers (Fig.10).

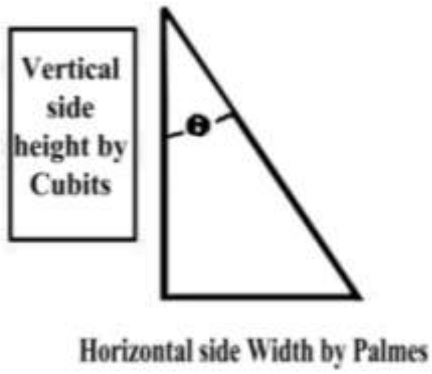

Fig10. *Diagram illustrating the equation of Seqed*

The tan (θ) is the value of the angle between the vertical side of height and the horizontal side of width. Then 7 tan (θ) is a result of the issue that 1 Royal cubit equals 7 palms = 28 fingers (or digits). The ancient Egyptians used similar triangles to measure dimensions and lengths of slopes, and they defined the seqed as the ratio of the run to the rise. Pyramid slopes were fixed in the designing stage of the building process via the tangent angle. Using the simplest and lowest possible numbers and fractions resulted in an approximation differing just 1% of the actual measured slopes of the pyramids. It is noteworthy that later sources such as the Rhind Papyrus from the First Intermediate Period, are dealing with issues like methods of measuring the tangent angle or seqed. The Rhind Papyrus discusses the tangent angles as obviously the best way of dealing with physical quantities **[41,43-44,95-99,100-102]**(Fig.11,a,b).

Fig11(a). *The Hieratic original discussing problems 56–60 of the Rhind Mathematical Papyrus (after: DE YOUNG, 2009; IMHAUSEN, 2016)*

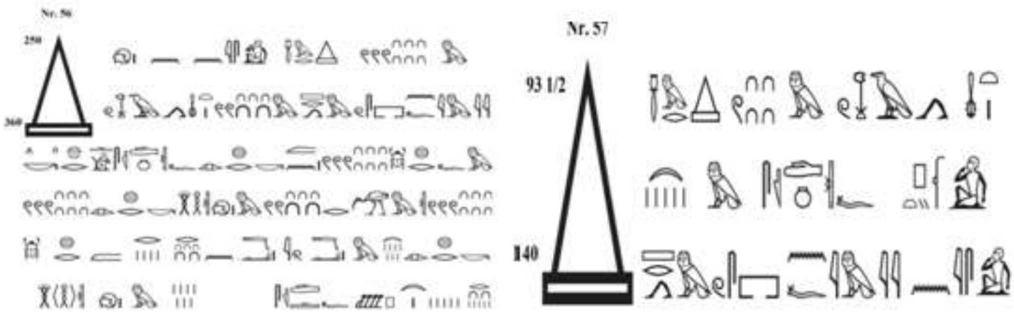

Fig11(b). *Hieroglyphic transcription of the problems Nos. 56–57 in the Rhind Mathematical Papyrus (after: EISENLOHR, 1877)*

Another mathematical papyrus discussing the tangent angle is the Moscow Mathematical Papyrus dated to the Middle Kingdom. It is also known as the GOLENISHCHEV Mathematical Papyrus after its first owner, Egyptologist VLADIMIR GOLENISHCHEV who bought the papyrus in 1892 or 1893 in Luxor. It later entered the collection of the Pushkin State Museum of Fine Arts in Moscow where it is kept today. This document too deals with the tangent angle called seqed **[42, 99,107]** defined as the number of palms and digits/fingers spanned horizontally for each cubit rise–or as ratio of the base (numerator) in cubits of a right triangle of one cubit in height. The Old Kingdom pyramid builders used the seqed successfully but during the Middle Kingdom errors may have occurred using this measuring method **[80, 91, 108]** (Fig.12).

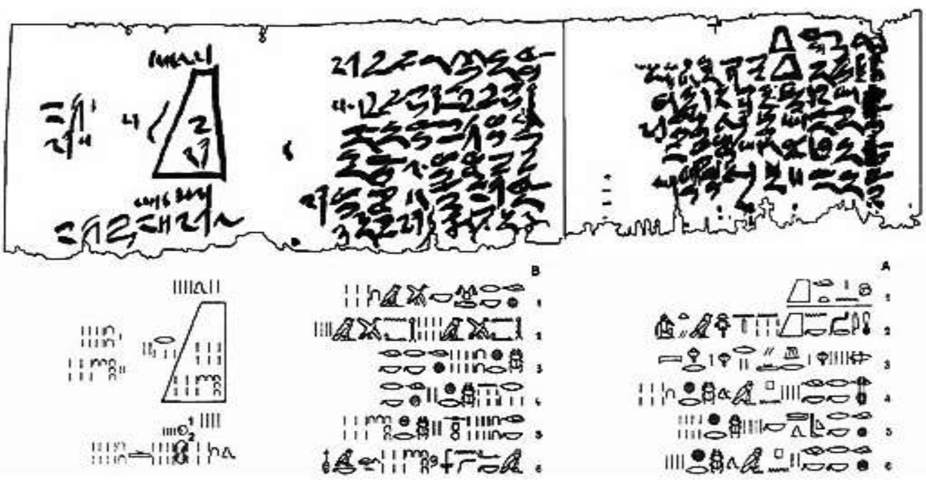

Fig12. *The 14th problem of the Moscow Mathematical Papyrus (after: STRUVE ,1930; CLAGETT,1999; http:// www.mathorigins.com/image%20grid.htm)*

Slope angles of pyramids of the Fourth and Fifth Dynasties were noticeably subject to some experimenting. Their sloping angles range between 6/5 and 4/3. In connection with this observation it has been suggested that angles were aligned to the sun, i.e., that the 4/3 triangle arose from measuring the zenith of the sun at culmination during the winter solstice, this measure being close to the complementary angle **[84, 108-115]**. It should be indicated that the height of the sun above the horizon at noon in the winter solstice $h_{min} = (90° – φ) – ε$, which is about 2500 years B.C. (during the Old Kingdom), where the inclination of the earth's axis to the equator – ε – was slightly less than 24°. The ratio of h_{min} ≈36° and the slope-angle should be Θ≈54.0° that gives a seqed 5.08 (5 1/11). The seqed of the Khufu's Pyramid is 5 ½, or θ≈51.8°. The difference of 2° is large enough to remain unnoticed. So the Pythagorean triangle probably originated from the surveying practice, in which the Egyptians were very sophisticated **[114]**. The occurrence of the phenomenon known as precession of the earth's axis of rotation, leading to a change in the direction of the axis in space, however the very precession has little in common with the value of the angle ε. Astronomers know that the Moon stabilizes the Earth's rotation in such a way that the variation of ε occurs in a very narrow range - only 2 degrees, with a period of about 41,000 years **[115]**. In more detail the earth rotates about an imaginary line called the axis of rotation which passes through the North and South Poles of the planet, in addition there are relationships between the zenith of the sun during the winter solstice with the axial precession and the orientation process of the astronomical bodies and Earth's rotational axis, therefore the orbit represents a symmetrical open curve (hyperbola) formed by the intersection of a plane with two identical cones on opposite sides of the same vertex. A hyperbola is concave toward North in the winter, concave toward South in the summer and a straight line at the Equinoxes. In connection with this the azimuth seems to be as the angle formed between specific directions from true north and a sight line of the observer to a specific point measured in the same scale as the provisional orthogonal direction to zenith as the highest point of the azimuth **[116-121]** (Fig. 13, a, b; 14, a, b).

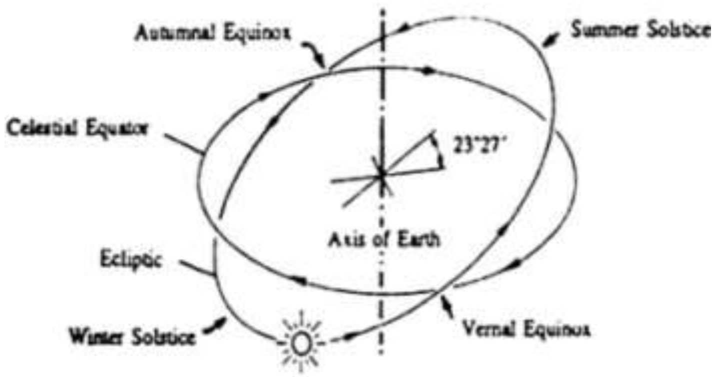

Fig13(a). *Diagram showing the axial precession and the orientation process of the astronomical bodies and Earth's rotational axis (after: Neugebauer, 1980; Isler, 1989).*

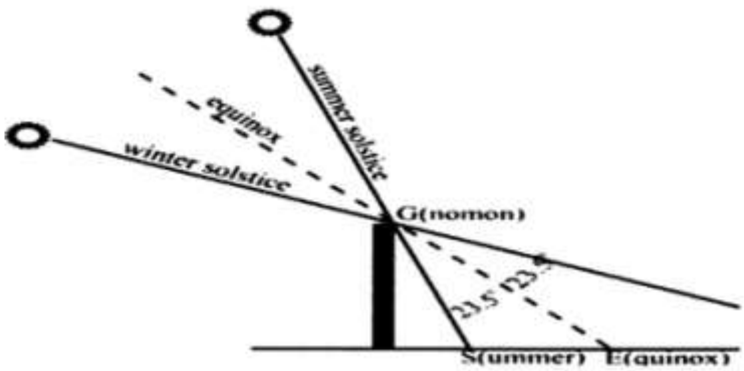

Fig13(b). *Diagram represents a symmetrical open curve (hyperbola) formed by the intersection of a plane with two identical cones on opposite sides of the same vertex. A hyperbola is concave toward North in the winter, concave toward South in the summer and a straight line at the Equinoxes (after: NEUGEBAUER,1980; ISLER,1989)*

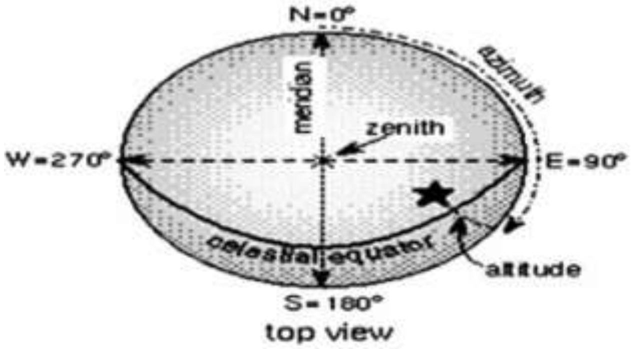

Fig14(a). *Diagram represents the azimuth that is the angle formed between specific directions from true north and a sight line of the observer to a specific point measured in the same scale as the provisional orthogonal direction to zenith as the highest point of the azimuth. The true north on the stereographic diagram is the positive Y axis 'straight up' and is marked with N (after: COUPRIE,2011; MAGLI,2013).*

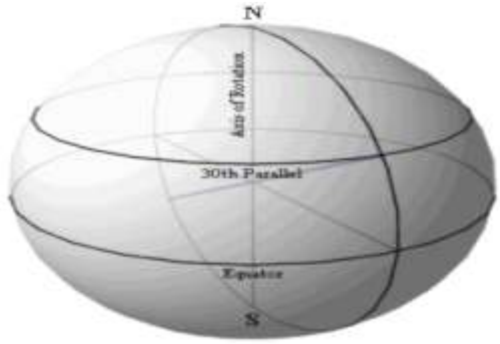

Fig14(b). *Diagram showing the process of the Earth's rotation. The earth rotates about an imaginary line called the axis of rotation which passes through the North and South Poles of the planet (after: WAKEFIELD, 2016).*

Noteworthy that (Fig.13, a) illustrates the momentary position of the earth's orbit (ecliptic) to the celestial equator with the indicated equinoxes and solstices. In addition to the precession of the axis of rotation (with a period of about 25800 years), the rotation of the apsid line of the earth's orbit with a period of about 21,000 years, so the result is a change in the duration of the seasons in a year **[116-120]**. Whilst (Fig.13, b) represents the diagram of the extreme altitudes of the sun at the solstices (with the figure being cut off from the side of the winter solstice), this is by Fixing the shadow points of the equinox and the solstices, and measuring the obliquity of the ecliptic with the help of a gnomon **[119-120]**. It is believed that the alignment and appropriate relative location of the so-called air-shafts of the King's Chamber in the Great Pyramid was determined by observing a celestial process now known as precession. PETRIE's survey in 1881 mentions that these shafts sloped southward in order to align with the position of the stars known as 'Delta Orionis' and northward in order to align with the stars known as 'Alpha Draconis' around 2600 B.C **[122]**. In 1990 PETRIE'S data were used to specify that the southern shaft of the Queen's Chamber sloped towards the constellation known as 'Sirius' around 2750 B.C**[123]**. In 1993, the German engineer RUDOLF GANTENBRINK was able to obtain more precise measurements for the angles of these shafts confirming these hypothetical astral orientations and dates (c. 2450 B.C. +/- 25 years) **[124]**. In 1995, the Scottish astronomer MARY BRUCK concurred with these findings but assumed a +/- 60 years margin of error **[125]**.

CONCLUSION AND ANALYSIS OF THE STUDY

The methodology and structure of this study clarified that there are different and dissonant values in measuring dimensions in ancient Egypt through the historical ages to the present days. Through the methodology and structure of this study, it can be said that there are some differences which have caused confusion when we want to measure the length of a cubit or to determine the accurate value of the seqed and inclination especially or dimensions in ancient Egypt generally. The length of the historical cubit varied in ancient times, where there were two main units; the first one measuring almost 18 inches and a second about 20 inches long. Also other variations occurred, some small-scale and some greater than usual. This may be due to very large deviations from the 'norm' at a given time. The concept and significance of the seqed in architecture becomes apparent when considering the inner slope or inclination of the triangular side of the pyramid. There, the seqed represents the run (or incline) which equals a vertical rise of 1 cubit, a word derived from the Latin ‘Cubitum’ for elbow (Greek πήχυς [pechys]). In terms of modern geometry a right triangle must be designed to determine the seqed, with triangle's vertical side representing the pyramids height and its horizontal side being half of the building's base-line. Then one should draw a comparable triangle whose vertical side is equal to 1 cubit. The seqed will then equal the length of the horizontal side of the second triangle. In modern trigonometric terms the seqed-to-the-height ratio (in ancient palms) is the cotangent angle of the triangular surface. In the Rhind Papyrus it is said that the value of the cubit equalled 7 palms and that the value of the seqed is expressed in palms and fingers (cf. specifically problem 59 a, but also problems 56, 57, 58, and 59 b). Problem 60 presents a different solution. Here the seqed is considered as a fraction of a cubit. In solution of problem 59 a, the ratio between the value called and 7 palms equals 1 cubit representing exactly the ratio of the half-side to the height. Simply speaking this, then, is the concept behind the term seqed. Also, this concept was suggested and considered in problems 56-60 of the Rhind Papyrus. The Egyptian seqed is the rise/run ratio of the slope of a cotangent broadly based on the ancient Egyptian measurements values of the Royal cubit, the palm and the digit. There are, however, some modifications which have caused confusion concerning the length of a cubit or the accurate value of a seqed. It should be mentioned that the confusion and the difficulties explaining the seqed problems arise in part from the technical words employed. EISENLOHR, the first editor of the Rhind Papyrus, mentions the different interpretations by scientists as early as 1877. EISENLOHR concluded that the term seqed referred to the ratio of two lines. Also, he concluded that the seqed should refer to either the ratio of the half-base to the apothem or to the ratio of the half-diagonal to the sharp edge formed by the meeting of two flat or curved surfaces. In ancient Egypt, there were seven palms in the cubit; in addition to the seqed was seven times the cotangent. The Egyptian seqed is the ratio of the run to the rise of a slope of the cotangent. The Rhind Papyrus which is an ancient Egyptian documentary source mentioned the seqed, which is the base of many problems or issues such as; 56, 57, 58, 59 ,59 b and 60. The Rhind Papyrus contains 84 mathematical problems to be solved and there is a section assigned to the orientation of pyramids, where the term ‘Seqed’ is used. The inclination of measured slopes was expressed as a value of horizontal palms and digits for every Royal cubit rise. This inclination a type of the contemporary measure, which related to ‘Gradient’, therefore it is a measure comparable to the slope angle of the current cotangent. The value of the seqed in ancient Egyptian architecture is differentiated; the slopes of the faces of the Great Pyramid of Giza were a seqed of 5 1/2, or 5 palms and 2 digits, which adjust to a slope of 51.84° from the horizontal, using the current 360 degree system. Technically, seqed or θ is using the current trigonometry, the cotangent of the angle between the base of the pyramid and one of its faces. Thus, if the tangent of an angle is a/b then the cotangent of that angle is b/a. It is noteworthy that the right triangle (ABS), where (AB) is the height and (BS) is half the length of the base. Calculating the seqed meant determining an angle θ. One could therefore say that also current trigonometry uses the seqed. In ancient Egyptian as well as in present trigonometry the cotangent of an angle is the value between the base of a pyramid and one of its faces. Thus, if the tangent of an angle is a/b then the cotangent of that angle is b/a. The seqed of a pyramid is considered to be the number of palms in the horizontal corresponding to a rise of one cubit or seven palms. The average width of the base of the Great Pyramid is about 370 ‘Horizon’ cubits (pechys baladi), the disparity is nearly 1 portion in 6,000 or 0.016%, and therefore the approximation number will supposedly be the planned or the meant width. This pyramid was apparently intended to conform to the proportions of a Pythagorean {3-4-5} triangle, giving a theoretical height of 246⅔ cubits or 370 feet. In fact, it is well known that the angle θ for the Pythagorean triangle is 53.13°, which is more than 1° greater than the angle for the Khufu's Pyramid.

RESULTS OF THE STUDY

- The methodology and structure of this study clarified that there are different and dissonant values in measuring dimensions in ancient Egypt through the historical ages to the present days. These differences caused confusion when we want to measure the length of a cubit or to determine the accurate value of the seqed and inclination especially or dimensions in ancient Egypt generally.

- The length of the historical cubit varied in ancient times, where there were two main units; the first one measuring almost 18 inches and a second about 20 inches long. Also other variations occurred, some small-scale and some greater than usual. This may be due to very large deviations from the 'norm' at a given time.
- In ancient Egyptian mathematics there were seven palms in a cubit, in addition to the seqed, which was seven times the cotangent. The Egyptian seqed is the ratio of the run to the rise of a slope of a cotangent. The Rhind Papyrus which is an ancient Egyptian mathematical document mentions the seqed repeatedly (e.g. 56, 57, 58, 59, 59 b and 60) in connection with many problems or issues.
- The concept and significance of the seqed in architecture becomes apparent when considering the inner slope or inclination of the triangular side of the pyramid. In modern trigonometric terms the seqed-to-the-height ratio (in ancient palms) is the cotangent angle of the triangular surface.
- The inclination of slopes was thus expressed as a value of horizontal palms and digits for every Royal cubit rise. The seqed represents the run (or incline) which equals a vertical rise of 1 cubit, a word derived from the Latin 'Cubitum' for elbow (Greek πήχυς [pechys]). Etymologically, the term 'pyk belady/baladi' is related to the Greek 'pekhus and pygon', both also meaning a cubit, with that being the number of cubits to the Greek stade (about 600 feet [180 meters]). Hence 'pyk belady' can be taken to mean 'national cubit [of Egypt]').
- The inclination of slopes was thus expressed as a value of horizontal palms and digits for every Royal cubit rise. This inclination may be a type of the contemporary measure belonging to the so- called Gradient which is in connection with the contemporary concept of cotangent and tangent. The value of the seqed in ancient Egyptian architecture varies therefore, where the slopes of the faces of the Great Pyramid of Giza measure a seqed of 5 1/2, or 5 palms and 2 digits, which amounts to a slope of 51.84°. In current trigonometry seqed (or θ) means the cotangent of the angle between the base of the pyramid and one of its faces. Thus, if the tangent of an angle is a/b then the cotangent of that angle is b/a. It is noteworthy that the right triangle (ABS), where (AB) is the height and (BS) is half the length of the base. The average width of the base of the Great Pyramid amounts to about 370 'Horizon' cubits (pechys baladi), the disparity is nearly 1 portion per 6,000 or 0.016%, and for our purpose we assume the approximate figure corresponding to the planned or desired width. Mathematical ratios existing in this pyramid seems to be symmetrical with the proportions or values of a Pythagorean {3-4-5} triangle, giving a theoretical height of 246 2⁄3 cubits or 370 feet. But in fact, it is well known that the angle θ for the Pythagorean triangle is 53.13°, which is more than 1° greater than the angle for the Khufu's Pyramid. Furthermore the seqed of a pyramid is calculated by finding x in terms of y then multiplying the coefficient of the determination of y by 7. Or the other way round: it is 7 times the cotangent of the pyramid's dihedral angle

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