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RESEARCH ARTICLE

On the Conical Pappus Spiral in a Gravitational Field

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Abstract

Many mathematical curves are old, but their meaning can be renewed when they are read in a new language. The conical Pappus spiral is such a curve. Classically, it can be regarded as an Archimedean spiral lifted onto a right circular cone, or as curve generated by simultaneous uniform rotation and uniform motion along the cone. Its horizontal projection is an Archimedean spiral, while suitable planar projections are associated with Doppler spirals. The curve is described by the cone angle α, the planar Archimedean spiral angle β, and the conical spiral angle γ. The speed formulae reveal β and γ as velocity-ratio angles. The acceleration properties of this curve emphasize a simple but useful result: for uniform angular motion the vertical velocity can be non-zero and constant, while the vertical acceleration is zero, so all acceleration is contained in the horizontal Archimedean projection. The aim of this paper is to model the motion of a quantum particle on this conical Pappus spiral in a gravitational field. The central result is the velocity interpretation: $\tan \gamma = v_{\varphi}/v_g$, where v_{φ} is the horizontal projection of the braking speed caused by the gravitational field and v_g is the speed along the cone generator. This turns an old equation into a compact physical language: alpha determines the cone ($\alpha = \pi/4$), beta determines the braking in the planar projection, gamma determines the winding of the spiral in a gravitational field. This model leads to a new interpretation of the formulae using a simple mathematical language in compare with a more complicated mathematical language known in the general theory of relativity.

Keywords: Archimedean Spiral, Conical Pappus Spiral, Formulae in the General Theory of Relativity, Velocity and Acceleration in a Gravitational Field.

1. Introduction: Old Equations and New Ways of Seeing[1]

The statement that "many good scientific ideas begin not as completely new equations, but as a new way of seeing old equations" is not a slogan against originality. It is a reminder that equations are not only algebraic objections: they are also carriers of interpretation. The same formula can be a static curve in one setting, a trajectory in another, a projection law in a third, and a physical metaphor in a fourth.

The conical Pappus spiral is a good example. Its basic equation is simple and classical. Yet when the curve is read as a motion on a cone, it reveals a clean velocity language. In that language, alpha fixes the cone, beta fixes braking in the planar projection, gamma fixes the balance between sliding along a generator and winding around the axis. The curve becomes a small laboratory in which geometry, kinematics, and interpretation meet.

The curve is historically associated with Pappus and later mathematicians, and modern mathematical sources describe its projections and its relation to Archimedean and Doppler spirals, e.g. [1]-[10]. The purpose here is different: to present a clear alpha-beta-gamma kinematic reading of formulae derived by general relativity for quantum particles in a gravitational field, e.g. [11]-[20].

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2. The Conical Pappus Spiral: Angles, Speeds, and Accelerations

The conical Pappus spiral can be read as a classical geometric curve, but it can also be read as a compact kinematic language. The notation α, β, and γ separates the cone geometry, the horizontal Archimedean projection, and the true conical winding. Fig. 1 shows these angles in this conical Pappus spiral.

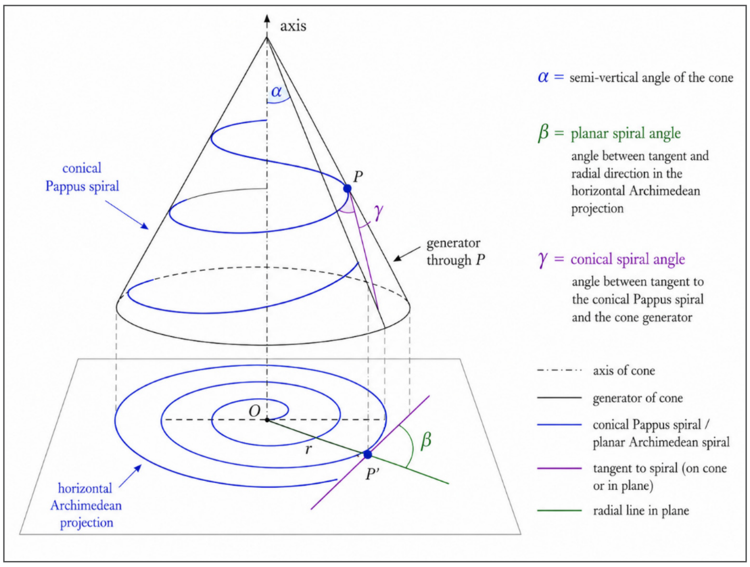

Figure 1. *The conical Pappus spiral with three characteristic angles: alpha (semi-vertical angle of the cone), beta (planar spiral angle), gamma (conical spiral angle).*

Table 1 Summarizes the interpretation of angles in the conical Pappus spiral

Table 1. *The characteristic angles in the conical Pappus spiral*

Symbol	Name	Definition	Revealing formula
α	semi-vertical cone axis	angle between cone axis and cone generator	$z = \rho \cot\alpha$
φ	azimuthal parameter	rotation angle around the cone axis	$\rho = a_r\varphi$ (a_r is the radial scale)
β	planar spiral angle	angle between tangent and radial direction in the horizontal Archimedean projection	$\tan\beta = \varphi$
γ	conical spiral angle	angle between tangent to the conical spiral and the cone generator	$\tan\gamma = \varphi \sin\alpha$
ψ	developed-cone angle	angular coordinate after cutting and flattening the cone	$\psi = \varphi \sin\alpha$
The bridge between β and γ is: $\tan\gamma = \sin\alpha \tan\beta$			

Fig. 2 shows the three characteristic velocities in the conical Pappus spiral: the planar (horizontal) velocity v_{plane}, the vertical velocity v_z, and the total 3D velocity v.

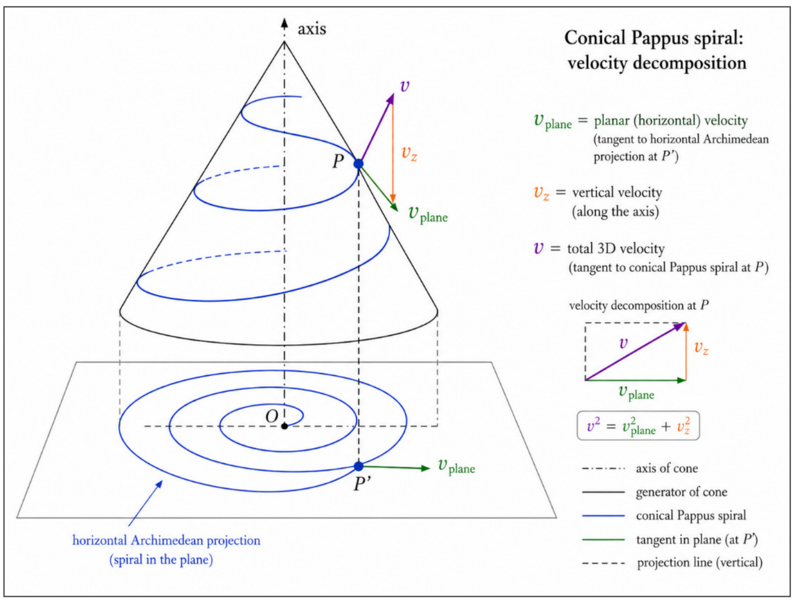

Figure 2. *The conical Pappus spiral with three characteristic velocities: vplane is the planar velocity, vz is the vertical velocity, and v is the total 3D velocity.*

Table 2 Summarizes the characteristic speeds in the conical Pappus spiral.

Table 2. *The characteristic speeds in the conical Pappus spiral*

Quantity	Formula	Meaning
radius	$\rho = a_r \varphi$	distance from the cone axis
height	$z = a_r \varphi \cot\alpha$	vertical coordinate
diameter	$D = 2 a_r \varphi$	diameter of the cone section
radial velocity	$v_\rho = a_r \dot{\varphi}$	signed radial component
azimuthal velocity	$v_\varphi = a_r \varphi \dot{\varphi}$	signed circular component
vertical velocity	$v_z = a_r \dot{\varphi} \cot\alpha$	signed vertical component
planar speed	$v_{plane} = a_r \lvert\dot{\varphi}\rvert \sqrt{1+\varphi^2}$	speed of the horizontal projection
generator speed	$v_g = a_r \lvert\dot{\varphi}\rvert \csc\alpha$	radial-vertical generator direction
3D speed	$v = a_r \lvert\dot{\varphi}\rvert \sqrt{\varphi^2 + \csc^2\alpha}$	total speed along the curve
3D speed with γ	$v = a_r \lvert\dot{\varphi}\rvert \csc\alpha \sec\gamma$	3D speed using conical angle
The planar spiral angle β compares azimuthal speed with radial speed $$\tan\beta = \frac{\lvert v_\varphi\rvert}{\lvert v_\rho\rvert}$$		
The conical spiral angle γ compares azimuthal speed with generator-direction speed $$\tan\gamma = \frac{\lvert v_\varphi\rvert}{\lvert v_g\rvert}$$		
Speed balance occurs $$\lvert v_\varphi\rvert = v_g \quad \text{or equivalently for } \gamma = 45^\circ$$		
The fractions of the total speed $$\frac{\lvert v_\rho\rvert}{v} = \sin\alpha\cos\gamma \quad \frac{\lvert v_\varphi\rvert}{v} = \sin\gamma \quad \frac{\lvert v_z\rvert}{v} = \cos\alpha\cos\gamma$$		
These three fractions of the total speed satisfy the check identity $$(\sin\alpha\cos\gamma)^2 + (\sin\gamma)^2 + (\cos\alpha\cos\gamma)^2 = 1$$		

Table 3 Summarizes the characteristic accelerations in the conical Pappus spiral.

Table 3. *The characteristic accelerations in the conical Pappus spiral for uniform angular motion*

Acceleration	Formula
For uniform angular motion	$\dot{\varphi} = \pm\Omega \qquad \ddot{\varphi} = 0$
The radial acceleration	$a_\rho = -a_r \varphi \Omega^2$
The azimuthal acceleration	$a_\varphi = 2 a_r \Omega^2$
The vertical acceleration	$a_z = 0$
The planar acceleration	$a_{plane} = a_r \Omega^2 \sqrt{\varphi^2+4} = a_r \Omega^2 \sqrt{\tan^2\beta + 4} = a_r \Omega^2 \sqrt{\frac{\tan^2\gamma}{\sin^2\alpha} + 4}$
The 3D acceleration	$a_{3D} = a_r \Omega^2 \sqrt{\varphi^2+4} = a_r \Omega^2 \sqrt{\tan^2\beta + 4} = a_r \Omega^2 \sqrt{\frac{\tan^2\gamma}{\sin^2\alpha} + 4}$
The ratio of the azimuthal acceleration to the radial acceleration magnitude	$\frac{\lvert a_\varphi\rvert}{\lvert a_\rho\rvert} = \frac{2}{\varphi} = \frac{2}{\tan\beta} = \frac{2\sin\alpha}{\tan\gamma}$
The acceleration-balance point occurs	$\lvert a_\varphi\rvert = \lvert a_\rho\rvert \quad when \quad \beta_{acc} = \arctan(2) \quad and \quad \gamma_{acc} = \arctan(2\sin\alpha)$

The conical Pappus spiral contains several interesting properties surveyed in Table 4.

Table 4. *The characteristic properties found in the conical Pappus spiral*

Observation	Meaning
Angle language	α determines the semi-vertical angle of the cone, β reads the planar winding, γ reads the true conical winding
Vertical acceleration simplicity	For uniform angular motion, v_z can be non-zero while $a_z = 0$
Horizontal acceleration laboratory	For uniform angular motion, all acceleration belongs to the horizontal Archimedean projection
Two balance radii	Velocity balance and acceleration balance occur at different positions on the spiral
Sign caution	Inward and outward motions reverse velocities, but uniform accelerations are unchanged in magnitude.

The conical Pappus spiral can be read as a classical geometric curve, but it can also be read as a compact kinematic language. The notation α, β, and γ separates the cone geometry, the horizontal Archimedean projection, and the true conical winding. The speed formulae reveal β and γ as velocity-ratio angles. The acceleration formulae reveal a further simplicity: under uniform angular motion, the vertical acceleration vanishes and the acceleration in entirely the acceleration of the planar Archimedean projection.

This suggests that the conical Pappus spiral is a useful study object for geometry, kinematics, constrained motion, surface dynamics, and projection effects. It is a good example of how an old curve can become newly useful when its formulae ae organized in a clearer physical language.

3. The Conical Pappus Spiral in a Gravitational Field

In this model we assume that a quantum particle is guided by the Planck intensity action h/2π [Js steradian^{-1}] on the helical path with the radius $a_r = \lambda/2\pi$ and the height of this helix z = λ (λ is the wavelength) and the helix angle α = 45°, [21]. This schema is shown in Fig. 3.

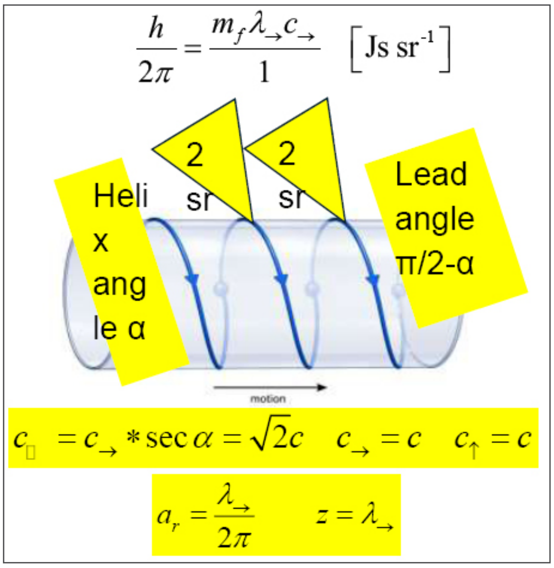

Figure 3. *The quantum particle moving on the helix guided by the Planck intensity action h/2π [Js steradian^{-1}].*

We insert this helix into a gravitational field that modifies this helix into the conical Pappus spiral. The gist of this model is the braking action of the gravitational field on the characteristic speeds of the conical Pappus spiral so we get the characteristic angles α, β, and γ as:

$$\alpha = 45^{\circ}$$

$$\tan\beta = \frac{v_{\varphi}}{v_{\rho}} = \frac{c\sqrt{\frac{2GM}{Rc^2}}}{c} = \sqrt{\frac{2GM}{Rc^2}}$$

$$\tan\gamma = \frac{v_{\varphi}}{v_g} = \frac{c\sqrt{\frac{2GM}{Rc^2}}}{\sqrt{2}c} = \sqrt{\frac{GM}{Rc^2}} \tag{1}$$

$$\tan\beta = \frac{\tan\gamma}{\sin\alpha}$$

Table 5 Summarizes the characteristic speeds of the conical Pappus spiral under the influence of the gravitational field.

Table 5. *The characteristic speeds found in the conical Pappus spiral inserted into the gravitational field*

Characteristic speed	Formula
radial rotation speed	$v_\rho = c$
radial braking speed	$v_\varphi = -c\sqrt{\frac{2GM}{Rc^2}}$
radial time dilation	$T_{radial} = -T_\infty\sqrt{\frac{2GM}{Rc^2}}$
plane speed	$v_{plane} = c\sqrt{1-\frac{2GM}{Rc^2}}$
plane time dilation	$T_{plane} = T_\infty\sqrt{1-\frac{2GM}{Rc^2}}$
vertical speed	$v_z = c = \lambda_\infty\sqrt{1-\frac{2GM}{Rc^2}}\,\frac{v_\infty}{\sqrt{1-\frac{2GM}{Rc^2}}}$
generator speed	$v_g = \sqrt{2}\,c$
3D speed	$v_{3D} = \sqrt{2}\,c\sqrt{1-\frac{GM}{Rc^2}}$
3D time dilation	$T_{3D} = \sqrt{2}\,T_\infty\sqrt{1-\frac{GM}{Rc^2}}$
Three fractions of the total speed satisfy the check identity $$\left(\frac{v_\rho}{v_{3D}}\right)^2 - \left(\frac{v_\varphi}{v_{3D}}\right)^2 + \left(\frac{v_z}{v_{3D}}\right)^2 = 1$$	
The interpretation of the Schwarzschild radius $$v_{plane} = 0 = c\sqrt{1-\frac{2GM}{Rc^2}} = c\sqrt{1-\frac{R_S}{R}}$$ $$m_{app} = \frac{m_{true}}{\left(1-\frac{R_S}{R}\right)}$$	
Comment: Max Abraham derived in 1911 his Eq. 6 in [22]-[25] $$\frac{c^2}{2} - \frac{c_0^2}{2} = \Phi - \Phi_0$$ c_0 is the speed of light at the origin, where the potential is Φ_0	

Table 6 Summarizes the characteristic accelerations of the conical Pappus spiral in the gravitational field for uniform angular motion.

Table 6. *The characteristic accelerations for uniform angular motion found in the conical Pappus spiral inserted into the gravitational field where* m_{app} *is the apparent mass and its observed effects*

Characteristic acceleration for the uniform angular motion	Formula
vertical acceleration	$a_z = 0$
planar acceleration	$a_{plane} = a_0 \left(1 - \frac{2GM}{Rc^2}\right)$
apparent mass of the quantum particle in the gravitation field	$m_{app} = \frac{F}{a} = \frac{m_{true}}{\left(1 - \frac{2GM}{Rc^2}\right)}$
$\frac{\Delta F}{F} = \frac{m_{app} - m_{true}}{m_{true}} = \frac{1}{\left(1 - \frac{2GM}{Rc^2}\right)} - 1 \approx \frac{2GM}{Rc^2}$	
The principle of equivalence $F = \frac{m_{true}}{\left(1 - \frac{2GM}{Rc^2}\right)} a = \frac{GM}{R^2} \frac{m_{true}}{\left(1 - \frac{2GM}{Rc^2}\right)}$	

The apparent mass of photons influences the photon bending angle δ around the Sun as it was introduced by Johann Georg von Soldner in 1804 [26] and Albert Einstein partly in 1911 [27] and fully in 1915 using his mathematical language in the general theory of relativity [11]-[14]:

$$\delta = \int_{\varepsilon=-\pi/2}^{\varepsilon=+\pi/2} \frac{2GM_{\odot}}{R_{\odot} C^2} \cos\varepsilon \, d\varepsilon = \frac{4GM_{\odot}}{R_{\odot} C^2} \qquad (2)$$

The symbol $M_{\odot}$ is the mass of the Sun and $R_{\odot}$ is the radius of the Sun.

4. Conclusion

The conical Pappus spiral is a classical curve with a modern didactic power. By assigning distinct meanings to alpha, beta, and gamma, the geometry becomes clearer: measures the opening of the cone, beta measures the winding of the horizontal Archimedean projection, and gamma measures the winding of the curve on the cone surface.

The speed formulae reveal the same structure dynamically. Beta compares azimuthal speed with radial speed, while gamma compares azimuthal speed with generator-direction speed. The characteristic speeds therefore provide a compact physical reading of the curve. The conical Pappus spiral is not only a geometric object; it is also a useful model for velocity decomposition on a developable surface.

The mathematical language of the conical Pappus spiral could be used to visualize the main formulae discovered by the abstract mathematical language of the general theory of relativity, e.g. [11]-[20]. The gist of this model is the braking action of the gravitational field on the speeds of a quantum particle moving on that conical Pappus spiral in the presence of the gravitational field. This suggest that the conical Pappus spiral is a useful study object for a visualization of events in the presence of the gravitational field.

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Conflict of Interest

The author declares that there is no conflict of interest.

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