

SHORT COMMUNICATION

A FLRW-Metric Universe with Zero Spatial Curvature is Incompatible with Dark Energy as a Cosmological Constant

Fernando Salmon Iza

Bachelor in Physics from the Madrid Complutense University UCM, Spain.

Received: 19 December 2025 Accepted: 05 January 2026 Published: 09 January 2026

Corresponding Authors: Fernando Salmon Iza, Bachelor in Physics from the Madrid Complutense University UCM, Spain, fernandosalmoniza@gmail.com.

Abstract

We have obtained a theoretical result that leads us to believe that a universe FLRW with zero spatial curvature and dark energy as cosmological constant is incompatible with the results corresponding to the acceleration of our universe over the last 6 billion years, a phenomenon attributed in the Λ CDM model to the existence of dark energy. To this end, we calculated, in the FLRW metric, the equation for the derivative of the Hubble parameter and studied its sign during the three eras of the universe: the radiation era, the matter era, and the dark energy era. We were able to verify that for zero spatial curvature and dark energy as the cosmological constant, a positive value for the derivative of the Hubble parameter will never occur; that is, the Hubble parameter, H , will never be increasing, which makes a period like the one occurring for the last 6 billion years, caused by dark energy, impossible.

Keywords: Dark Energy, Spatial Curvature, Friedmann Equations, Cosmology.

1. Introduction

Given the Friedmann equations, [1], of the FLRW metric, (ρ = Joules/m³).

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3c^2}(\rho + 3p) + \frac{\Lambda c^2}{3}$$

1.1 Equation that Relates the Derivative of the Hubble Parameter with Pressure, Energy Density and the Curvature Spatial

According to the Friedmann equation, the equation of state, and taking into account the expression for the derivative of H , we have:

$$H' = (\dot{a}/a)' = (\ddot{a}/a) - (\dot{a}/a)^2$$

$$H' = \left(-\frac{4\pi G}{3c^2}(\rho + 3p) + \frac{\Lambda c^2}{3}\right) - \left(\frac{8\pi G\rho}{3c^2} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}\right)$$

$$H' = -\frac{12\pi G(\rho + p)}{3c^2} + \frac{kc^2}{a^2}$$

equation of state: $w = p/\rho$

$$H' = -\frac{12\pi G\rho(1+w)}{3c^2} + \frac{kc^2}{a^2}$$

1.2 Study of H During the Universe Eras Using this Equation

Let's examine the sign of this equation in the different eras that make up the history of the universe. That is, during the radiation era, the matter era, and the dark energy era, which is the current one. Each of these eras is characterized by a different value for the parameter of the equation of state, w ($w = +1/3$ in the period of universe dominated by radiation, $w = 0$ in the period of universe dominated by matter, $w = -1$ in the period of universe dominated by dark energy). Furthermore, according to experimental data [2], the term $\Omega_k = (kc^2)/a^2$ is currently very small, and its sign is still

Citation: Fernando Salmon Iza. A FLRW-Metric Universe with Zero Spatial Curvature is Incompatible with Dark Energy as a Cosmological Constant. Open Access Journal of Physics. 2026; 8(1): 01- 02.

©The Author(s) 2026. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

undetermined. In addition, a change in trend has been observed in the value of the Hubble parameter, H , as a function of the age of the universe, increasing since 6 billion years ago.[3]. Our study concludes that this is not possible with the FLRW metric, which has zero spatial curvature and dark energy as the cosmological constant.

We summarize these calculations and reasoning in the following tables.

Considering that the curvature term of the Friedmann equation is small compared to the other terms of the equation, the following will be fulfilled.

Period of the universe dominated by radiation, from its beginning until the year 360,000

$$w = +1/3$$

$$H' = -\frac{12\pi G\rho(1+\frac{1}{3})}{3c^2} + \frac{kc^2}{a^2}$$

For $k = 0, +1, -1$. H' will be negative. H will always be decreasing.

Period of the universe dominated by matter, from year 360,000 to 6 billion years ago

$$w = 0$$

$$H' = -\frac{12\pi G\rho}{3c^2} + \frac{kc^2}{a^2}$$

For $k = 0, +1, -1$. H' will be negative. H will always be decreasing.

Period of the universe dominated by dark energy, from 6 billion years ago until its end

$$w = -1$$

$$H' = \frac{kc^2}{a^2}$$

For $k = -1$, H' will be negative, then H will be decreasing.

For $k = 0$, H' will be zero then H will be constant.

For $k = +1$, H' will be positive, then H will be increasing.

Our universe, according to the results of Mission Planck[2], has zero or very near- zero spatial curvature; in the scenario we are analyzing in this work, its curvature is zero. It is clear from our discussion that

for zero spatial curvature, the universe can never have a positive acceleration that would increase the Hubble parameter, at least under our assumption of considering dark energy in the Friedmann equations solely as a cosmological constant. It is therefore necessary to add other terms to the Friedmann equations that can represent dark energy and make it possible for the Hubble parameter to increase during the era of the universe dominated by dark energy, that is, from 6 billion years ago [3].

2. Conclusion

Using an equation that represents the value of the derivative of the Hubble parameter in an FLRW metric and dark energy as the cosmological constant, we have analyzed the sign of this derivative as a function of the three eras that make up the universe and as a function of the space curvature, $k = 0, +1, -1$. This has led us to conclude that for zero space curvature, as is the case in our current universe according to the results of Mission Planck [2], the Friedmann equations never predict an increase in the Hubble parameter like the one observed [3], experimentally in our universe for the last 6 billion years due to dark energy. This leads us to believe that the term we are using in the Friedmann equations to represent dark energy is not the correct one. Thus, from a theoretical point of view, we also encourage a thorough study of dark energy with new hypotheses that lead to its correct inclusion in the Friedmann equations.

3. References

1. Weinberg Stephen. Gravitation and cosmology; principles and applications of the general theory of relativity. John Wiley and Sons. Inc New York, London, Sidney, Toronto. 1972.
2. Planck, 2018 results. Cosmological parameters. Astronomy & Astrophysics; Vol 641(67). 2020.
3. Riess A. G. et al. Observational evidence from supernova from an accelerating universe and cosmological constant. The Astronomical Journal. 116(3), 1009- 10038. 1998.