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RESEARCH ARTICLE

Common Fixed Point and Coincidence Point for three Singled-Valued Mappings in Metric Spaces using Rational Expressions

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Abstract

This paper aims to prove some coincidence point results for three single-valued mappings in the setting of metric spaces and partially ordered metrics spaces satisfying a generalized contraction condition of rational type. These contributions extend the existing literature on three single-valued mappings and fixed point theory. We showcase the practical applicability of our proposed notions

Keywords: Common Fixed Point, Rational Contractions, Metric Space, Partially Ordered Metric Space.

1. Introduction

The classical Banach contraction principle (BCP) [14] is one of the most notable results which has played a vital role in the development of a metric fixed point theory.The principle has been generalizedby numerous authors in the different directions by improving the underlying contractionconditions, enhancing the number of involved mappings, weakening theinvolved metrical notions, and enlarging the class of ambient spaces [2, 3, 7, 8, 10, 11,13].

In 2004,Ran and Reurings [16] obtained a new variant of the classical Banach contraction principle toa complete metric space endowed with partial order relation, which was slightly modifiedby Nieto and Rodriguez-Lopez [12] in 2005 and established fixed point results.

Recently, Mehmood [6] obtained somecommon fixed point results for generalized rational typesatisfying compatible three self mappings in complex valued b-metric space

Based on the above insight, we introduce the concepts of three single-valued mappings and subsequently establish common and coincidence point results for partially ordered metrics spaces satisfying a generalized contraction condition of rational type within the setting of metric spaces. To bolster our findings, we showcase the practical applicability of our proposed notions.

2. Preliminaries

Here we begin with the following definitions that are relevant in our study.

Definition 1.1.[7, 15]Assume$(X,\preccurlyeq)$ is a partially ordered set, then $T:X\to X.T$ is referred to as monotone non-decreasing if for all $x,y\in X$,

$x\preccurlyeq y$ implies $Tx\preccurlyeq Ty$.

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Definition 1.2.[1]Suppose $(X, \preccurlyeq)$ is a partially ordered set and $T: g, h: X \to X$ are mappings such that $T(X) \subseteq h(X)$ and $g(X) \subseteq h(X)$. Then T and g are weakly increasing with respect to h if and only if for all $x \in X$, we have

a) $Tx \leq gx$ for all $y \in h^{-1}(Tx)$,

b) $gx \leq Ty$ for all $y \in h^{-1}(gx)$.

Definition 1.3.[4, 5, 17]Let S be a nonempty subset of a metric space (X, d) and $T, g : S \to S$. A point $x \in S$ is a common fixed (respectively, coincidence) point of g and T if $x = gx = Tx$ (respectively $gx = Tx$). The set of fixed points (respectively, coincidence points) of g and T is denoted by $F(g, T)$ (respectively, $C(g, T)$).

The pair (T, g) is called

a) commutative if $Tgx = gTx$ for all $x \in S$;

b) compatible if $\lim d(Tgx_n, gTx_n) = 0$ whenever $\{x_n\}$ is a sequence such that $\lim Tx_n = \lim gx_n = t$ for some t in S;

c) weakly compatible if g and T commute at their coincidence points, i.e. if $gTx = Tgx$ whenever $gx = Tx$.

3. Main Results

Theorem 3.1. Let S be a subset of a metric space (X, d). Suppose that $T, g, h : S \to S$ satisfy

$$d(Tx, gy) \leq \alpha\left(\frac{d(hx, Tx)d(hy, gy)}{d(hx, hy) + d(hx, gy) + d(hy, Tx)}\right) + \beta\left(\frac{d(hx, Tx)d(hy, gy)}{d(hx, hy)}\right) + \gamma(d(hx, hy)) \quad (3.1)$$

for all $x, y \in S, hx \neq hy$ and for some $\alpha, \beta, \gamma \in [0, 1)$ with $\alpha + \beta + \gamma < 1$.

Suppose also that $T(S) \cup g(S) \subseteq h(S)$ and $(h(S), d)$ is complete. Then

(i) T, g and h have a coincidence point in S;

(ii) If the pairs (h, T) and (h, g) are weakly compatible, then T, g and h have a unique common fixed point.

Proof. Suppose $x_0 \in X$. Since $T(S) \cup g(S) \subseteq h(S)$, we choose $x_1, x_2 \in S$ so that $hx_1 = Tx_0$ and $hx_2 = gx_1$. By induction, we construct a sequence $\{x_n\}$ in X such that $hx_{2n+1} = Tx_{2n}$ and $hx_{2n+2} = gx_{2n+1}$ for every $n \geq 0$. By (3.1), we have

$$d(hx_{2n+2}, hx_{2n+1}) = d(gx_{2n+1}, Tx_{2n})$$
$$\leq \alpha\left(\frac{d(hx_{2n+1}, gx_{2n+1})d(hx_{2n}, Tx_{2n})}{d(hx_{2n+1}, hx_{2n}) + d(hx_{2n+1}, Tx_{2n}) + d(hx_{2n}, gx_{2n+1})}\right) + \beta\left(\frac{d(hx_{2n+1}, gx_{2n+1})d(hx_{2n}, Tx_{2n})}{d(hx_{2n+1}, hx_{2n})}\right) + \gamma(d(hx_{2n+1}, hx_{2n}))$$
$$= \alpha\left(\frac{d(hx_{2n+1}, hx_{2n+2})d(hx_{2n}, hx_{2n+1})}{d(hx_{2n+1}, hx_{2n}) + d(hx_{2n+1}, hx_{2n+1}) + d(hx_{2n}, hx_{2n+2})}\right) + \beta\left(\frac{d(hx_{2n+1}, hx_{2n+2})d(hx_{2n}, hx_{2n+1})}{d(hx_{2n+1}, hx_{2n})}\right) + \gamma(d(hx_{2n+1}, hx_{2n}))$$
$$\leq \alpha\left(\frac{d(hx_{2n+1}, hx_{2n+2})d(hx_{2n}, hx_{2n+1})}{d(hx_{2n+1}, hx_{2n}) + d(hx_{2n}, hx_{2n+2})}\right) + \beta\left(\frac{d(hx_{2n+1}, hx_{2n+2})d(hx_{2n}, hx_{2n+1})}{d(hx_{2n+1}, hx_{2n})}\right) + \gamma(d(hx_{2n+1}, hx_{2n}))$$

By triangular inequality $d(hx_{2n+1}, hx_{2n+2}) \leq d(hx_{2n+1}, hx_{2n}) + d(hx_{2n}, hx_{2n+2})$, we have

$$= \alpha(d(hx_{2n}, hx_{2n+1})) + \beta(d(hx_{2n+1}, hx_{2n+2})) + \gamma(d(hx_{2n+1}, hx_{2n})) \quad (3.2)$$

which implies that

$$d(hx_{2n+2}, hx_{2n+1}) \leq \left(\frac{\alpha+\gamma}{1-\beta}\right) d(hx_{2n+1}, hx_{2n}) \quad (3.3)$$

By mathematical induction, we have

$$d(hx_{2n+2}, hx_{2n+1}) \leq \left(\frac{\alpha+\gamma}{1-\beta}\right)^{2n+1} d(hx_{2n+1}, hx_{2n}) \quad (3.4)$$

where $r = \frac{\alpha+\gamma}{1-\beta} < 1$. We now need to prove that $\{hx_n\}$ is a Cauchy sequence. For $m \geq n$, we have

$$d(hx_m, hx_n) \leq d(hx_m, hx_{m-1}) + d(hx_{m-1}, hx_{m-2}) + \cdots + d(hx_{2n+1}, hx_n)$$
$$\leq (r^{m-1} + r^{m-2} + \cdots + r^n) d(hx_1, hx_0)$$
$$\leq \left(\frac{r^n}{1-r}\right) d(hx_1, hx_0) \quad (3.5)$$

implies that $d(hx_1, hx_0) \to 0$ as $m, n \to \infty$. Hence $\{hx_n\}$ is a Cauchy sequence.

Since $(h(S), d)$ is complete, there exists $t \in S$ such that $hx_n \to ht$ as $n \to \infty$.

We now prove that t is a coincidence point of T, g and h. We have

$$d(hx_{2n+1}, gt) = d(Tx_{2n}, gt)$$
$$\leq \alpha\left(\frac{d(hx_{2n}, Tx_{2n})d(ht, gt)}{d(hx_{2n}, ht) + d(hx_{2n}, gt) + d(ht, Tx_{2n})}\right) + \beta\left(\frac{d(hx_{2n}, Tx_{2n})d(ht, gt)}{d(hx_{2n}, ht)}\right) + \gamma(d(hx_{2n}, ht))$$
$$= \alpha\left(\frac{d(hx_{2n}, hx_{2n+1})d(ht, gt)}{d(hx_{2n}, ht) + d(hx_{2n}, gt) + d(ht, hx_{2n+1})}\right) + \beta\left(\frac{d(hx_{2n}, hx_{2n+1})d(ht, gt)}{d(hx_{2n}, ht)}\right) + \gamma(d(hx_{2n}, ht))$$

Letting $n \to \infty$, we get $d(ht, gt) = 0$, hence $ht = gt$. Also, we have

$$d(Tt, ht) = d(Tt, gt)$$
$$\leq \alpha\left(\frac{d(ht, Tt)d(ht, gt)}{d(ht, ht) + d(ht, gt) + d(ht, Tt)}\right) + \beta\left(\frac{d(ht, Tt)d(ht, gt)}{d(ht, ht)}\right) + \gamma(d(ht, ht))$$

implies that $d(Tt, ht) = 0$, that is, $Tt = ht$. Thus, $ht = Tt = gt$.

Hence t is a coincidence point of T, g and h.

Now, suppose that the pairs (h,T) and (h,g) are weakly compatible. Let $u = gt = ht = Tt$. Then we have $hTt = Tht$ and $hgt = ght$, which implies that $Tu = gu = hu$. Also, we can have

$$d(hu,u) = d(Tu,gt) \leq \alpha\left(\frac{d(hu,Tu)d(ht,\ gt)}{d(hu,ht)+d(hu,gt)+d(ht,Tu)}\right)+\beta\left(\frac{d(hu,Tu)d(ht,gt)}{d(hu,ht)}\right) + \gamma(d(hu,ht))$$

This implies that $d(hu,u) = 0$, that is $hu = u$. Hence, we get

$u = hu = Tu = gu,$

a common fixed point of h, T and g.

Now, suppose that $u' \in S$is another common fixed point of h, T and g, that is

$u' = hu' = Tu' = gu'$.

Then, we have

$$d(u,u') = d(Tu,gu') \leq \alpha\left(\frac{d(hu,Tu)d(hu',gu')}{d(hu,hu')+d(hu,gu')+d(hu',Tu)}\right)+\beta\left(\frac{d(hu,Tu)d(hu',gu')}{d(hu,hu')}\right) + \gamma(d(hu,hu')).$$

Implies $d(u,u') = 0$, that is $u = u'$. Thus, we have the uniqueness of the common fixed point.

Now, we prove the existence and uniqueness ofcommon fixed point theorem in the context of metric space endowed with partial order.

Theorem 3.2. Let $(X,d,\preccurlyeq)$ be a complete partially ordered metric space. Suppose that$T, g, h : X \to X$ satisfy

$$d(Tx,gy) \leq \alpha\left(\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)+d(hx,gy)+d(hy,Tx)}\right)+\beta\left(\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)}\right) + \gamma(d(hx,hy))(3.6)$$

for all $x, y \in X, hx \preccurlyeq hy$ and for some $\alpha, \beta, \gamma \in [0,1)$ with $\alpha + \beta + \gamma < 1$.

Suppose that

(a) $T(X) \subseteq h(X), g(X) \subseteq h(X)$ and$h(X)$ is a complete subspace of X;

(b) T and g are weakly increasing with respect to h.

Also suppose that either

(i) the pairs $(T,h)$is compatible and$T, h$are continuous; or

(ii) the pairs $(g,h)$is compatible and $g, h$are continuous.

Then, T, g and h have a coincidence point, that is, there exists t in X such that

$ht = gt = Tt$.

Proof. Let $x_0 \in X$. From (a) we can choose $x_1, x_2 \in X$ such that $hx_1 = Tx_0$ and $hx_2 = gx_1$. By induction, we construct a sequence $\{hx_n\}$ in X such that $hx_{2n+1} = Tx_{2n}$ and $hx_{2n+2} = gx_{2n+1}$ for every $n \geq 0$.

We claim that

$$hx_n \preccurlyeq hx_{n+1}, \text{ for all } n \geq 1. \quad (3.7)$$

Since T and g are weakly increasing mappings with respect to h, we get

$hx_1 = Tx_0 \preccurlyeq gy, \forall y \in h^{-1}(Tx_0)$.

Since $hx_1 = Tx_0$, then $x_1 \in h^{-1}(Tx_0)$, and we get

$hx_1 = Tx_0 \preccurlyeq gx_1 = hx_2$.

Again,

$hx_2 = gx_1 \preccurlyeq Ty, \quad \forall y \in h^{-1}(gx_1)$.

Since $hx_2 = gx_1$, then $x_2 \in h^{-1}(gx_1)$, and we get

$hx_2 = gx_1 \preccurlyeq Tx_2 = hx_3$.

By induction on n, we have

$hx_1 \preccurlyeq hx_2 \preccurlyeq \cdots \preccurlyeq hx_{2n+1} \preccurlyeq hx_{2n+2} \preccurlyeq \cdots$

implies claim (3.7) holds.

Since $hx_{2n} \preccurlyeq hx_{2n+1}$, for all $n \geq 1$, from (3.6), we have

$$d(hx_{2n+2},hx_{2n+1}) = d(gx_{2n+1},Tx_{2n}) \leq \alpha\left(\frac{d(hx_{2n+1},gx_{2n+1})d(hx_{2n},\ Tx_{2n})}{d(hx_{2n+1},hx_{2n})+d(hx_{2n+1},Tx_{2n})+d(hx_{2n},gx_{2n+1})}\right) + \beta\left(\frac{d(hx_{2n+1},gx_{2n+1})d(hx_{2n},Tx_{2n})}{d(hx_{2n+1},hx_{2n})}\right)+\delta(d(hx_{2n+1},hx_{2n})) + \gamma(d(hx_{2n+1},hx_{2n}))$$

$$= \alpha\left(\frac{d(hx_{2n+1},\ hx_{2n+2})d(hx_{2n},\ hx_{2n+1})}{d(hx_{2n+1},hx_{2n})+d(hx_{2n+1},hx_{2n+1})+d(hx_{2n},hx_{2n+2})}\right) + \beta\left(\frac{d(hx_{2n+1},hx_{2n+2})d(hx_{2n},hx_{2n+1})}{d(hx_{2n+1},hx_{2n})}\right)+\gamma(d(hx_{2n+1},hx_{2n}))$$

$$\leq \alpha\left(\frac{d(hx_{2n+1},\ hx_{2n+2})d(hx_{2n},\ hx_{2n+1})}{d(hx_{2n+1},hx_{2n})+d(hx_{2n},hx_{2n+2})}\right)+\beta\left(\frac{d(hx_{2n+1},hx_{2n+2})d(hx_{2n},hx_{2n+1})}{d(hx_{2n+1},hx_{2n})}\right) + \gamma(d(hx_{2n+1},hx_{2n}))$$

By triangular inequality

$d(hx_{2n+1},\ hx_{2n+2}) \leq d(hx_{2n+1},hx_{2n}) + d(hx_{2n},hx_{2n+2})$

$d(hx_{2n+1},\ hx_{2n+2}) \leq d(hx_{2n+1},hx_{2n}) + d(hx_{2n},hx_{2n+2})$

, we have

$$= \alpha(d(hx_{2n+1},hx_{2n})) + \beta(d(hx_{2n+1},hx_{2n+2})) + \gamma(d(hx_{2n+1},hx_{2n}))(3.8)$$

which implies that

$$d(hx_{2n+2},hx_{2n+1}) \leq \left(\frac{\alpha+\gamma}{1-\beta}\right)d(hx_{2n+1},hx_{2n})(3.9)$$

By mathematical induction, we have

$$d(hx_{2n+2},hx_{2n+1}) \leq \left(\frac{\alpha+\gamma}{1-\beta}\right)^{2n+1} d(hx_{2n+1},hx_{2n})(3.10)$$

where $r = \frac{\alpha+\gamma}{1-\beta} < 1$. We now need to prove that $\{x_n\}$ is a Cauchy sequence. For $m \geq n$, we have

$$d(hx_m, hx_n) \leq d(hx_m, hx_{m-1}) + d(hx_{m-1}, hx_{m-2}) + \cdots + d(hx_{2n+1}, hx_n)$$

$$\leq (r^{m-1} + r^{m-2} + \cdots + r^n) d(hx_1, hx_0)$$

$$\leq \left(\frac{r^n}{1-r}\right) d(hx_1, hx_0) (3.11)$$

implies that $d(hx_1, hx_0) \to 0$ as $m, n \to \infty$. Hence $\{hx_n\}$ is a Cauchy sequence.

Since $(h(X), d)$ is complete, there exists $t \in X$ such that $hx_n \to ht$ as $n \to \infty$.

Suppose that condition (i) holds. Let $u = ht$. Then we have

$$\lim_{n\to\infty} Tx_{2n} = \lim_{n\to\infty} hx_{2n} = u.$$

Since the pair (T, h) is compatible, we get

$$\lim_{n\to\infty} d(h(Tx_{2n}), T(hx_{2n})) = 0. \qquad (3.12)$$

From the continuity of T and h, we have

$$\lim_{n\to\infty} d(h(Tx_{2n}), T(hx_{2n})) = d(hu, Tu). (3.13)$$

Using (3.12), (3.13) and by the uniqueness of the limit, we have $d(hu, Tu) = 0$, that is $hu = Tu$. Using (3.6), we get

$$d(hu, gu) = d(Tu, gu)$$
$$\leq \alpha\left(\frac{d(hu, Tu)d(hu, gu)}{d(hu, hu) + d(hu, gu) + d(hu, Tu)}\right) + \beta\left(\frac{d(hu, Tu)d(hu, gu)}{d(hu, hu)}\right)$$
$$+ \gamma(d(hu, hu)),$$

implies that $d(hu, gu) = 0$, that is, $hu = gu$. Thus, $hu = gu = Tu$.

Hence u is a coincidence point of T, h and g.

If condition (ii) holds, then the same argument follows, we get result.

4. Applications

Some applications of the main results to a self mapping involving an integral type contraction.

Let us consider the set of all functions χ defined on $[0, \infty)$ satisfying the following conditions:

1. Each χ is Lebesque integrable mapping on each compact subset of $[0, \infty)$.
2. For any $\epsilon > 0$, we have $\int_0^\epsilon \chi(t)dt > 0$.

Theorem4.1 Let S be a subset of a metric space (X, d). Suppose that $T, g, h : S \to S$ satisfy

$$\int_0^{d(Tx,gy)} \phi(t)dt$$
$$\leq \alpha \int_0^{\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)+d(hx,gy)+d(hy,Tx)}} \phi(t)dt$$
$$+ \beta \int_0^{\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)}} \phi(t)dt + \gamma \int_0^{d(hx,hy)} \phi(t)dt \ (4.1)$$

for all $x, y \in S$ and for some $\alpha, \beta, \gamma \in [0,1)$ with $\alpha + \beta + \gamma < 1$.

Suppose also that $T(S) \cup g(S) \subseteq h(S)$ and $(h(S), d)$ is complete. Then

(i) T, g and h have a coincidence point in S;

(ii) If the pairs (h, T) and (h, g) are weakly compatible, then T, g and h have a unique common fixed point.

Theorem4.2 Let $(X, d, \preccurlyeq)$ be a complete partially ordered metric space. Suppose that $T, g, h : X \to X$ satisfy

$$\int_0^{d(Tx,gy)} \phi(t)dt$$
$$\leq \alpha \int_0^{\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)+d(hx,gy)+d(hy,Tx)}} \phi(t)dt$$
$$+ \beta \int_0^{\frac{d(hx,Tx)d(hy,gy)}{d(hx,hy)}} \phi(t)dt + \gamma \int_0^{d(hx,hy)} \phi(t)dt \ (4.2)$$

for all $x, y \in S$ and for some $\alpha, \beta, \gamma \in [0,1)$ with $\alpha + \beta + \gamma < 1$.

Suppose that

(a) $T(X) \subseteq h(X)$, $g(X) \subseteq h(X)$ and $h(X)$ is a complete subspace of X;

(b) T and g are weakly increasing with respect to h.

Also suppose that either

(i) the pairs (T, h) is compatible and T, h are continuous; or

(ii) the pairs (g, h) is compatible and g, h are continuous.

Then, T, g and h have a coincidence point, that is, there exists t in X such that

$$ht = gt = Tt.$$

5. Conclusion

The main findings of this study demonstrate applicability for three single-valued mappings in establishingcommon and coincidence point theorems for partially ordered metrics spaces satisfying a generalized contraction condition of rational type. This study provides significant advancements in the understanding of metric spaces, with potential applications in differential equations.

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Conflicting Interests

There are no competing interests, according to the authors.

Author's Contributions

All authors contributed equally to the writing of this paper.

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