

RESEARCH ARTICLE

A General Theory of Gravity with Applications to Determining the Mutually Attractive Forces between Moving Masses, the Center of Gravity, and the Resolution of the Dark Matter Problem

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Abstract

While Newton's Law of Gravity is often a useful starting point in the study of gravity, it is shown to be in error when it is employed to find forces when masses are close to each other or when the long-term effects concerning orbiting bodies are under study. In its stead an alternate law is proposed which avoids these errors. This law is then used to explain the strange behavior of stars in dark matter studies. The proposed law also provides an equation for calculating the center of gravity of a given mass which can differ from the center of mass. A simple example is used to prove this point.

1. Introduction

In this study Newton's Law of Gravity (NLOG) will be shown to be adequately accurate in certain situations, but not in others. Accordingly, a general law of gravity (GLOG) will be proposed which eliminates these errors. It will be further shown that the proposed theory leads to a determination of the center of gravity which usually differs from the center of mass. GLOG will then be used to explain the strange movements of stars in galaxies, so that there is probably no dark matter in the universe.

2. Newton's Law of Gravity (NLOG)

Assume that masses m_1 and m_2 are stationary with respect to a given inertial frame of reference, IFR₀, and *r* is the instantaneous distance between them. Then from Newton's Law of Gravity (NLOG) the mutually attractive gravitational force *f* exerted by each mass on the other is as follows:

2.1 Newton's Law of Gravity (NLOG)

(2.1)
$$f = G m_1 m_2 / r^2$$

In this equation G is called the universal gravitational

constant. While it has been estimated in the lab that $G \approx 6.672 x 10^{-11} m^3 / kgs^2$, there are theoretical problems associated with the experiments that find G which will be covered in this work.

3. Problems with NLOG

One of the reasons Newton set forth his equation was to evaluate the attractive forces exerted by orbiting bodies on one another. In these cases the distances between the bodies are large. As a general rule, NLOG is an approximation which works reasonably well in these situations, as long as the time spans under study are not too great. This is the case, for example, in Kepler's law. However, especially when the distances between the masses are short or when the time spans under study are very large, then there are situations when various factors must be taken into account, some of which are as follows:

(a) Is the center of gravity (cg) equal to the center of mass (cm)?

(b) If not, then how do you find it?

(c) What happens when at least one of the masses is not stationary?

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(d) How do you deal with arbitrary mass shapes?

The General Law of Gravity (GLOG) proposed in this work will handle these difficult issues.

4. A Barbell Thought Experiment

Whenever the center of gravity (cg) happens to be calculated, it is generally assumed to be at the center of mass (cm). Sometimes this assumption is based on torque analysis when the gravitational acceleration is constant. In the simple barbell thought experiment given here it will be shown that it is possible that the cg is not at the cm.

Consider a given inertial frame of reference, IFR_o, and assume that two masses, m_1 and m_2 , are stationary within IFR₀, where $m_1 = m_2 = m/2$. In order to avoid problems which will become evident in the theory developed later in this work, assume that these masses are infinitesimally small. Let the two dimensional (x, y)positions of m_1 and m_2 be as follows: $(x_1 = c, y_1 = 0)$ and $(x_2=-c, y_2=0)$. Thus, the masses are positioned in a barbell arrangement on equal sides of the x axis at y=0 in the xy plane. Next, assume an observer is on the y axis in the xy plane at $r_{OBS} = (0, Y)$, where Y > 0. As $m_1 + m_2$ will be viewed as a single mass of total value *m*, where $m=m_1+m_2$, assume in this thought experiment that these two masses are attached by a weightless rod to form a barbell. In this example the cm of the barbell is at $\mathbf{r}_{cm} = (0, 0)$.

It is argued in this work that NLOG is a valid law when the two masses are stationary and differentially small in size, which applies in this case. Thus, the total force f exerted by the barbell on an arbitrarily differentially small mass, M_{OBS} , situated at the observation point, $r_{OBS} = (0, Y)$, has a net value in the $-\mu_y$ direction, where μ_y is a unit vector on the y axis. Since it is assumed that gravitational forces are additive, then f is the sum of the individual forces due to m_1 and m_2 on the observer mass M_{OBS} as viewed at (0, Y). Thus, f is found as follows:

(4.1)
$$\mathbf{f} - [Gm_1 M_{OBS} \cos(\theta_1)/D_1^2 + Gm_2 M_{OBS} \cos(\theta_2)/D_2^2] \boldsymbol{\mu}_{v}$$

In the above, D_i is the distance from m_i to the observation point, as follows:

$$(4.2) D_1^2 = D_2^2 = c^2 + Y^2$$

Since $D_1 = D_2$, define D from (4.2) as follows:

$$(4.3) D^2 = D_i^2 = c^2 + Y^2$$

It is noted that the minus sign appears in (4.1) because f is an attractive force. Since the force components

cancel in the direction orthogonal to μ_y , the $cos(\theta_i)$ terms in (4.1) reflect the fraction of the force in the $-\mu_y$ direction, and they are given as follows:

(4.4)
$$cos(\theta_{\gamma}) = cos(\theta_{\gamma}) = Y/D$$

Accordingly, from these comments (4.1) reduces to the following

(4.5)
$$f = - [G m M_{OBS} Y / D^3] \mu_v$$

Now consider what happens if the total barbell mass of m is instead located as a fixed point mass at the center of gravity, \mathbf{r}_{cg} . The objective here is to determine \mathbf{r}_{cg} so that the force exerted on M_{OBS} by the point mass m is the same as the value of f as given by (4.5). In this particular thought experiment the barbell weights fall symmetrically about the y axis, so that it the cg falls on the y axis. In particular, suppose it falls on the y axis at a distance L in the - $\boldsymbol{\mu}_y$ direction from the observation point at $\mathbf{r}_{OBS} = (0, Y)$. Thus, if y_0 is the y component of \mathbf{r}_{ce} , then $y_0 = Y-L$ and the following obtains:

(4.6)
$$\mathbf{r}_{cg} = (0, y_0) = (0, Y - L)$$

If, for example, the cg happened to be at the cm, then $y_0=0$ and from (4.6) L=Y. However, it turns out this is not the case. *L* is found by equating the actual f at $r_{OBS}=(0,Y)$ to the value that would be achieved if the entire mass of *m* were a single point mass located at $r_{cg}=(0,Y-L)$. Thus, from (4.5) and the assumption that the force between two point masses is proportional to the product of the masses and $1/D^2$, the following obtains:

(4.7) $f = - [GmM_{OBS} Y/D^3] \mu_y = - [GmM_{OBS}/L^2] \mu_y$ From (4.7) it is seen that $Y/D^3 = 1/L^2$, so that:

$$(4.8) L^2 = D^3 / Y$$

Since D > Y, it is seen from (4.8) that $L^2 < L^3/Y$, so that L/Y > I and therefore:

Thus, from (4.9) it is concluded that the cg, which is at $\mathbf{r}_{cg} = (0, y_0) = (0, Y-L)$, satisfies the following:

$$(4.10) \ y_0 = Y - L < 0$$

Therefore, the cg is not at the cm in this example.

Based on this relatively simple thought experiment the following conclusions are drawn:

4.1 Experiment Conclusions

(a) The center of mass is not necessarily at the center of gravity.

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(b) The cg will generally depend on the position of the observer.

(c) The cg is not necessarily just a single point.

5. The Differential Law of Gravity (DLOG)

In this section the differential law of gravity (DLOG) will be proposed. It will be the fundamental law upon which the General Law of Gravity (GLOG) in the next section will be based. In contrast to NLOG, assume that both m_1 and m_2 are differentially small in size, so that the notation used for these masses will be dm_1 and dm_2 . Also assume that either or both of these masses may be non-stationary. It is therefore seen that this model is quite general for two masses, except for the differential mass sizes. Next, suppose the user has a given fixed inertial frame of reference, say IFR₀. For this given IFR_0 , assume that IFR(t) is the instantaneous inertial frame of reference of dm_1 at time t, so that dm_1 is temporarily stationary in IFR(t) at this instant. A key assumption in DLOG is that a gravitational ray is continuously sent out from dm_1 (and also from dm_2) which travels at the speed of light, c, with respect to the instantaneous inertial frame of reference, IFR(t), of m_1 . Actually, what is sent out is a gravitational field that is a sphere, but calling it a ray better fixes attention on the upcoming analysis. This situation is similar to a moving fighter plane that fires a missile at time twhich travels at a given speed relative to the inertial frame of reference of the plane at the instant of firing. The missile subsequently hits the target at some later time, $t+\Delta t$. It is noted that the assumption concerning the movement of gravitational forces is in line with experimental results concerning the gravitational waves that have been detected following cataclysmic events in outer space.

As measured in IFR(t), assume that $\mathbf{R}(t) = R(t)\mathbf{u}(t)$ is the distance vector the ray travels until it hits dm₂ at some later time, $t+\Delta t$, where dm₂ may have been moving during the time Δt , and where $\mathbf{u}(t)$ is a unit vector in the direction of $\mathbf{R}(t)$. It is noted that $\mathbf{R}(t)$ may be different from the initial line-of-sight vector, $\mathbf{R}_0(t) = R_0(t)\mathbf{u}_0(t)$. Assume at the moment of impact that the velocity of m_2 is $V_2 = V_2\mathbf{u}_2$, where \mathbf{u}_2 is a unit vector, all as measured in IFR(t). Also, assume that m_2 is moving at an angle of φ with respect to the arriving ray, so that $cos(\varphi) = \mathbf{u}(t) \cdot \mathbf{u}_2(t)$. Thus, the relevant m_2 speed, V_{REL} , with respect to the direction of the ray is given by $V_{REL} = V_2 \cos(\varphi)$, all as measured in IFR(t). Now define Z as follows:

(5.1) $Z = V_2 \cos(\varphi) / c = V_{REL} / c$

From (5.1) it is seen that Z=1 when dm_2 is moving in the direction of u(t) at a velocity of c. Conversely, Z=-1 when dm_2 is moving at a velocity of c in the direction of -u(t). Thus, $-1 \le Z \le 1$.

A critical assumption will now be made concerning the gravitational force the ray exerts on dm_2 . It is argued that this force linearly decreases when the V_2 velocity lines up with the ray velocity, and v.v. Moreover, if it were to happen that that $V_{REL}/c=1$, it is argued there would be no force at all. Conversely, if $V_{REL}/c=-1$, it is argued the exerted force would be doubled from its value when both masses happen to be stationary. With this linearity assumption these arguments reduce to the following law for the differential force, df, exerted by the ray on dm_2 :

(5.2)
$$df = -df_0 (1-Z) = -df_0 (1-Z) u(t)$$

As it is argued that Z in real world applications is generally extremely small, so that the linearity assumption of Z in (5.2) is very local. In this equation df_0 is based on the spirit of Newton's Law as follows:

(5.3)
$$df_0 = G dm_1 dm_2 / R^2(t)$$

It is noted that $df_0 > 0$ in (5.3), so that df in (5.2) is in the -u(t) direction when $1-Z \ge 0$, which is presumably always the case. As the gravitational force is attractive, the result concerning the negative sign in (5.2) is intuitively obvious. It is also seen from (5.3) that df_0 is identical to NLOG when the masses are differential except that R(t) is not the line-of-sight value.

Accordingly, from (5.2) and (5.3) the differential law of gravity (DLOG) is given as follows:

Differential Law of Gravity (Dlog)

(5.4) $df = -df_0 (1-Z) = -(1-Z) u(t) G dm_1 dm_2 / R^2(t)$

It is seen from (5.4) that there are differences between DLOG and NLOG, as follows:

(a) In DLOG the gravitational force travels at c, whereas NLOG does not consider such a movement.

(b) $\mathbf{R}(t)$ is not necessarily the original line of sight vector.

(c) The exerted force takes into consideration the relative motion of m_2 with respect to the ray movement at the time of arrival.

(d) All calculations are based on the instantaneous inertial frame of reference, IFR(t) at the emission time *t*.

(e) If either dm_1 or dm_2 or both are nonstationary in IFR₀, then probably $Z \neq 0$, but Z is very small.

(f) If Z=0, DLOG becomes NLOG except for the dm's.

It is noted that the results of the thought experiment in Section 4 are in agreement with DLOG.

6. The General Law of Gravity (GLOG)

Concerning the problem of finding the gravitational attractive force f exerted by m_1 and m_2 on each other, it is assumed that this force may be found by integrating df as given by (5.4) over all dm_1 in m_1 and all dm_2 in m_2 . Thus:

The General Law of Gravity (Glog)

(6.1)
$$f = \iint df = - \iint (1 - Z) u(t) G dm_1 dm_2 / R^2(t)$$

In this equation R(t) is the distance the ray travels as it moves from dm_1 to dm_2 . Thus, more strictly, $R(t)=R(t,dm_1,dm_2)$. In the interpretation of (6.1) each differentially small element of mass dm_1 exerts a differentially small mutually attractive force df on each small element of mass dm_2 , and v.v. Unfortunately, performing the above integration may not be an easy matter, even in the case when both masses are constant density spheres.

7. Finding The Center of Gravity (cg)

Part of the problem of determining the position, r_{cg} , of the center of gravity of a given stationary mass, m, involves the definition what the cg means. It is assumed here that m may have an arbitrary shape and that its mass density may not be constant. It is argued that r_{cg} should depend on the position, r_{OBS} , of the observer. In this regard, assume an arbitrary stationary differential mass of dM_{OBS} is at the observer, which is at r_{OBS} , and that m exerts a force, f_{OBS} , on dM_{OBS} . As both m and dM_{OBS} are assumed to be stationary in a given IFR, then Z=0 in DLOG and GLOG. Also, from DLOG it is clear that the magnitude of f_{OBS} is proportional to both dM_{OBS} and m. Thus, f_{OBS} can be written in the following form:

(7.1)
$$\boldsymbol{f}_{OBS} = [G \ m \ dM_{OBS} / L^2] \boldsymbol{u} = \alpha \boldsymbol{u}$$

In (7.1) \boldsymbol{u} is a unit vector which points in the direction from \boldsymbol{r}_{OBS} to some point in the interior of mass m, so that $\alpha = [G \ m \ dM_{OBS} / L^2] > 0$. Also, the distance Land both α and \boldsymbol{u} are assumed to be known from an application of GLOG. Thus, L in (7.1) is known and is determined from the following:

(7.2)
$$L^2 = [G \ m \ dM_{OBS}] / \alpha$$

Now, instead of the actual mass *m* being distributed in some arbitrary way, assume it is totally situated at a given point, \mathbf{r}_{cg} , which will be called the center of gravity. Then, by definition of the cg, the force \mathbf{f}_{cg} exerted by this point mass m on dM_{OBS} at \mathbf{r}_{OBS} satisfies the following:

(7.3)
$$f_{cg} = f_{OB}$$

As both *m* and dM_{OBS} are point masses, then $f_{cg} = [G m dM_{OBS}/L_2] u$, so from the above equations the force f_{cg} exerted by a point mass *m* on a point mass dM_{OBS} is determined as follows:

(7.4)
$$\mathbf{f}_{c\sigma} = [G \ m \ dM_{OBS} / L^2] \ \mathbf{u} = \alpha \ \mathbf{u}$$

Since the cg is at a distance L along the vector u from the observation point, the following obtains:

(7.5)
$$r_{cg} = r_{OBS} + L u$$

As the values in (7.5) are known, then r_{cg} is determined. It is noted that the r_{cg} vector as given by (7.5) is the solution of the thought experiment given in Section 4. Also, it is clear from the above analysis and the thought experiment that r_{cg} is not necessarily at the center of mass, or even that the line drawn from the observation point through the center of mass necessarily passes through the center of gravity.

8. Application: When m₁ is the Earth

One of the major applications of gravitational theory occurs when m_1 is the earth and the size of m_2 is small compared the earth. It is assumed in the calculations that both m_1 and m_2 are stationary, so that Z=0 in DLOG and GLOG. Based on the huge size of the earth's radius and the diminutive size of m_2 , it is assumed that the cg of any combination of dm_2 and the earth is virtually a constant for all dm_2 in m_2 . Also, since m_1 is essentially infinitely greater than m_2 , and since the cg of the total mass, m_1+m_2 , is hugely far away from all dm_2 , the gravitational force exerted by m_1 on m_2 is virtually constant for all dm_2 in m_2 . Thus, it can be concluded that the force, df, exerted by m_1 on any dm_2 satisfies the following equation:

(8.1)
$$df = (constant) (dm_{\gamma}) (u)$$

In (8.1) \boldsymbol{u} is a constant unit vector for all dm_2 in m_2 . Thus, on determining \boldsymbol{f} by integrating over all dm_2 in (6.1), the result is as follows:

(8.2)
$$f = (constant) (m_2) (u) = m_2 g$$

In (8.2) g is a vector which varies slightly with the position and elevation of m_2 on the earth's surface. It is noted that g, which is termed the acceleration due

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to gravity, almost precisely points to the center of the earth, and that (8.2) agrees with actual experimental tests. The standard value for the magnitude of g is 9.80665 m/s². It is concluded that the use of (8.2) is a good formula for determining the gravitational force exerted by the earth on any mass m_2 .

Though u very closely points to the center of the earth, the cg is not actually at this center because L as given in the prior analyses is not necessarily the radius of the earth. This would be the case even if the earth were a perfect sphere with a constant mass density.

9. Situations When GLOG->NLOG

From (5.7) the two requirements for GLOG to be equal to NLOG are that Z=0 (i.e., both masses m_1 and m_2 are stationary) and they must be miniscule. However, in the case when the masses are orbiting bodies, it is argued the two bodies may be considered to be single points even though the mass sizes may be huge. This is because all radii vectors drawn from any dm_1 in m_1 to any dm_2 in m_2 are virtually identical. From this result it is seen that Newton was basically correct in his NLOG formula, except that

 $Z \neq 0$. Also, it is argued that the Z effect in GLOG of a ray when it hits m_2 is generally very small. Thus, NLOG can be used for orbiting bodies, so long as the time span under study is not too large.

As will be discussed in the next section, the forces that orbiting stars in a galaxy exert on each other do not have Z=0, so that over long time spans the Z effect matters.

10. Application of GLOG to the Dark Matter Problem

A key NLOG error in evaluating the mutually attractive force that a gravitational field exerts from one moving celestial body with mass m_1 on another moving body with mass m_2 is that the theory neglects the Z effect in GLOG. Also, the actual distance vector of the ray is not the line-of-sight vector. Even though Z is probably very small, it is argued that it may significantly change the motion and position of a mass if the time span under study is huge.

This situation is studied in Aucamp[1], where the gravitational model assumed has a quadratic Z function in V_{REL}/c with undetermined coefficients, rather than the linear model used here in GLOG. Also, the model does not use differential masses as in DLOG. Though the force exerted by one mass on another is therefore a little different from that of GLOG, nevertheless there

is enough similarity between the two models to argue that the strange star motions explained in that paper can likewise be explained by GLOG. As the analysis is quite extensive, it will be omitted here. However, the conclusion remains the same, in that GLOG explains why there probably is no dark matter in the universe

11. Application of GLOG to Constant Density Spheres

The two problems which need to be solved concerning constant density spheres are:

a) Problem #1: The find the cg of a single sphere when the observer is somewhere outside it.

b) Problem #2: Find the mutually attractive force between m_1 and m_2 , when both m_1 and m_2 are constant density spheres, but not necessarily identical ones.

The solution of either of these problems using GLOG is easier said than done. The mathematics of finding f by solving (6.1) through integral analysis can be complicated, at least in the case when the distances involved are not great. The solution of both of these problems is now under current study.

It is noted that two constant density small spheres are often used in experiments to determine G. These "Cavendish" tests measure the mutually attractive force exerted by each sphere on the other, where the spheres are not necessarily identical. Unfortunately, as NLOG is used for the force model and not GLOG, it is concluded there may be an error in the calculation of G.

12. Final Conclusions

Newton's Law of Gravity (NLOG) asserts that the mutually attractive force between masses m_1 and m_2 separated by a distance r is given by $f = Gm_1m_2/r^2$. It is argued that this model is valid if two conditions, C_1 and C_2 , are met, which are as follows:

(C₁) Both m_1 and m_2 are differentially small point masses.

(C₂) Both m_1 and m_2 are stationary.

Depending on the particulars, the failure of either one of these conditions can lead to problems.

The Differential Law of Gravity (DLOG) proposed in this work solves the gravity problem even when the two masses are not necessarily stationary, but are differentially small in size. Then, based on DLOG, the General Law of Gravity (GLOG) is formulated by integrating over all dm_1 in m_1 and all dm_2 in m_2 . From GLOG the cg of a given mass can be calculated, and it is shown that:

(a) The cg depends on the position of the observer.

(b) The cg is not necessarily at the cm.

These points are demonstrated in a simple barbell thought experiment.

It is assumed in DLOG that each differential mass sends out a continuous gravitational ray (actually, the ray is spherical) which travels at the velocity of light in its instantaneous inertial frame of reference. When the ray from, say, dm_1 arrives at the future position of dm_2 , it exerts a force on dm_2 which in part depends on the relative velocity V_{REL} of dm_2 in the direction of the movement of the ray.

In the case when m_i is the earth, it is shown that the gravitational force is given by f=mg. Thus, even though the center of gravity may not be known, the gravitational force can still be calculated.

By way of note, in Aucamp[1] a simplistic version of GLOG is assumed, and by extensive analysis it is concluded that the strange movements of stars can be explained, so it is quite probable there is no dark matter in the universe. As the analysis in that paper is too detailed to be summarized here, it is nevertheless argued that the theory is close enough to GLOG in certain respects that the derived conclusions are the same. Also, in Aucamp[2] the model in Aucamp[1] was used in a lesser role with other theory to explain the appearance of dark energy in the universe.

Since Einstein's General Theory of Relativity resolves none of the problems resolved in this work, such as the gravitational forces that moving masses have on each other, the determination of the center of gravity and the resolution of the dark matter issue, it is argued that GLOG should replace Einstein's theory.

13. References

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