

RESEARCH ARTICLE

Proofs of the Law of Atomic Radii and the Atomic Numbers of the Noble Elements

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Abstract

In a recently published paper in OAJP this author proposed a model of the atom in which orbiting electrons are arranged in strings in such a way that they do not radiate when in steady state. The analysis assumed the following two laws: (a) the law of atomic radii and (b) the law of the atomic numbers of the noble elements. While it is clear these laws are correct as based on the derived analyses and experimental evidence, it is not otherwise intuitively clear why they must be valid. To this end they are formally proved herein.

1. Introduction

This is the final paper in a series of three works which deals with atomic structure (Aucamp[1]), quantum theory (Aucamp[2]), and radiation (Aucamp[2]). In [1] a Law of Electrons (LOE) is proposed wherein electrons are viewed as single, string-like fields which are attached together in overlapping ring doublets in such a way that the electric fields they create when orbiting in steady state are constant in space, so that there is no radiation. The analysis in [1] leads to several extreme regularities concerning the structure of the periodic table. In that analysis two laws are temporarily assumed to be valid. These laws are now proved in this work. Specifically, they are the Law of Noble Elements (LNE) and the Law of Radii (LOR).

2. The Atomic Numbers of the Noble Elements

It is experimentally clear that the atomic numbers $\{Z_n^*\}$ of the seven noble elements as given by (2.1) below are correct, where single asterisks are used to denote true values. These elements are inherently inert because all their orbits are in some way fully packed. They represent the atoms on the extreme right side of the periodic table. From this table their atomic numbers are as follows:

Law of Noble Elements (LNE)

$$(2.1) \quad \{Z_n^*\} = \{2, 10, 18, 36, 54, 86, 118\}$$

These values refer to helium ($Z_1^*=2$), neon ($Z_2^*=10$), argon ($Z_3^*=18$), krypton ($Z_4^*=36$), xenon ($Z_5^*=54$), radon ($Z_6^*=86$), and oganesson ($Z_7^*=118$). It is noted that little is known of oganesson, which is extremely unstable. From the principal of maximum packing, it is clear that the only candidate atomic numbers $\{Z_n\}$ under consideration must have the property that $Z_{n+1} > Z_n$ for all $n < 7$.

While the atomic numbers of the noble elements are known, it is not understood why they must be given as in (2.1). For example, why must noble element #2 (neon) have an atomic number of $Z_2^*=10$ electrons? As this atom has two electrons in its inner orbit and eight in its outer orbit, why is this arrangement so stable?

In Aucamp[1] the actual structure of the periodic table was assumed as given by LNE. In addition, the Law of Radii (LOR) as given by (2.2) below was also assumed to be valid and then used to prove beyond any reasonable doubt this author's Law of Electrons (LOE). Accordingly, the purpose of this work is to prove that both LNE and LOR are valid, over and

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above their convincing applications in Aucamp[1]. In this section LOR will temporarily be assumed as given and then proved in the following section. In (2.2) the radius r_n of the n^{th} atomic ring is given by LOR as follows:

Law of Radii (LOR)

$$(2.2) r_n = n^2 r_0.$$

If (2.1) is correct, then the true number of electrons, S_n^* , in orbit n is found from $S_{n+1}^* = Z_{n+1}^* - Z_n^*$, so that $\{S_n^*\}$ is as follows:

$$(2.3) \{S_n^*\} = \{2, 8, 8, 18, 18, 32, 32\}$$

Based on LOE as given in Aucamp[1], electrons only exist in pairs, so that the number of pairs N_n^* in the n^{th} outer orbit is $N_n^* = S_n^*/2$. Thus:

$$(2.4) \{N_n^*\} = \{S_n^*/2\} = \{1, 4, 4, 9, 9, 16, 16\} = \{1^2, 2^2, 2^2, 3^2, 3^2, 4^2, 4^2\}$$

As these values are all perfect squares, define $J_n^* = (N_n^*)^{1/2}$. Then:

$$(2.5) \{J_n^*\} = \{(N_n^*)^{1/2}\} = \{1, 2, 2, 3, 3, 4, 4\}$$

Finally, if $\delta_n^* = J_n^* - J_{n-1}^*$ for $n > 1$, then from (2.5):

$$(2.6) \{\delta_n^*\} = \{1, 0, 1, 0, 1, 0\}$$

In (2.6) the value of δ_1^* is not defined because J_{-1}^* has no meaning. From (2.3), (2.4), (2.5) and (2.6) it is seen there is extreme regularity in all these series. Since they are all derived from the LOE theory proposed in Aucamp[1], this regularity is a strong endorsement of it. As all of the above equations assumed that both LNE and LOR were as given by (2.1) and (2.2), the objective of this work will be to prove the validity of these two laws. One immediate point in their favor is that they lead to the extremely convincing results as given above.

From (2.3) it is seen that $S_{n+1}^* \geq S_n^*$. This result stems from the fact that $r_{n+1} > r_n$, so from the principle that the noble elements are maximally packed it is clear that more and more electrons can be packed into an orbit as r_n increases with n . Thus, if $\{S_n\}$ is an arbitrary trial solution under consideration, then the following is a requirement for all n :

$$(2.7) S_{n+1} \geq S_n$$

In Aucamp[1] the minimum electron length is $L = 2\pi r_0$. Thus, from (2.2) the maximum number of electrons, S_n^{**} , that can be packed into the n^{th} orbit with radius $r_n = n^2 r_0$ is given as $2[2\pi r_n / L_n]$, where the factor of 2 is used because the electrons exist as doublet strings. Therefore:

$$(2.8) S_n^{**} = 2 [2\pi r_n] / (2\pi r_0) = 2n^2$$

From these result:

$$(2.9) \{S_n^{**}\} = \{2, 8, 18, 32, 50, 72, 98\}$$

One might assume that the true values as given by S_n^* should equal the maximum pack values as given by S_n^{**} . However, from (2.3) it is seen this is not the case.

Since electrons appear in doublets, then the number of pairs, N_n^{**} , indicated by S_n^{**} is as follows:

$$(2.10) \{N_n^{**}\} = \{S_n^{**}/2\} = \{1, 4, 9, 16, 25, 36, 49\}$$

As all the N_n^{**} are perfect squares, then J_n^{**} can be defined in a manner similar to the definition of J_n^* in (2.5), as follows:

$$(2.11) \{J_n^{**}\} = \{[N_n^{**}]^{1/2}\} = \{1, 2, 3, 4, 5, 6, 7\}$$

From the above results it clear that $\{N_n^*\} \neq \{N_n^{**}\}$ and $\{J_n^*\} \neq \{J_n^{**}\}$. Accordingly, the major question to be answered in this work is why is this so? The answer to this question will be found by examining what happens when an electron pair gets knocked out of its orbit. In particular, the solution will be based on examining what happens after such an ejection from the n^{th} orbit. While this orbit can theoretically hold $S_n^{**} = 2n^2$ electron pairs, it turns out that there can be a problem in replacing an ejected pair.

In this proof concerning the derivation of $\{Z_n^*\}$, it is sufficient to find either $\{S_n^*\}$ or $\{N_n^*\}$ or $\{J_n^*\}$. To this end the following three rules, R_1 , R_2 , and R_3 , dealing with the values of J_n which can be considered as candidates for J_n^* will temporarily be assumed and then subsequently proved.

Rule 1 (R1)

$$(2.12) J_n = (N_n)^{1/2} = (S_n/2)^{1/2} = \text{integer}$$

Rule 2 (R2)

$$(2.13) J_n = J_{n+1} = J_{n+2} \Rightarrow \text{non-feasible solution}$$

Rule 3 (R3)

$$(2.14) J_{n+2} = J_{n+1} + 1 = J_n + 2 \Rightarrow \text{non-feasible solution}$$

It is noted that R_2 does not allow three consecutive J_n values which are equal and R_3 does not allow three consecutive J_n values which increase by unity.

Based on the above three rules it is easy to show that the only possible candidate $\{J_n\}$ solution is $\{J_n^*\}$, so that the actual $\{Z_n^*\}$ found in nature must be as given by (2.1). First, since exactly one pair can fit into the r_1 orbit, then $J_1 = J_1^* = 1$ is the only possible solution. Next, $J_2^* = J_2^{**} = 2$ because this is the maximum pack

solution and it satisfies (R_1, R_2, R_3) . Accordingly, the problem begins when $n > 2$. Starting with $J_1^* = 1$ and $J_2^* = 2$, then from R_3 it is required that $J_3 = J_3^* = 2$. Then, from R_2 it is required that $J_4^* = J_3^* + 1 = 3$. Similarly, on moving along in this manner for the remaining values involving $4 \leq n \leq 7$, it is clear that the only admissible values for J_n are $J_n = J_n^*$, where J_n^* is as given in (2.5). Thus, the only feasible $\{Z_n\}$ candidate solution is $\{Z_n^*\}$. Therefore, in conclusion, if rules R_1 , R_2 and R_3 are correct, then the problem concerning $\{Z_n^*\}$ is solved.

Temporarily assuming R_1 is correct, the proofs that both R_2 and R_3 are correct will now be given. First, it will be shown that R_2 is correct. For a counter example to this rule, suppose that $J_2 = J_3 = J_4$, so that $S_2 = S_3 = S_4$. Further suppose that a doublet emission happens to the $n=3$ orbit. Then $S_3 < S_2$ and electrical feasibility requires a doublet replacement move from $n=4$ to $n=3$. This move results in $S_4 < S_3$, which is a non-feasible situation. Thus, it is concluded that $J_n = J_{n+1} = J_{n+2}$ is not a feasible candidate solution, so that rule R_2 is correct.

Next, it will be shown that R_3 is correct, once again assuming R_1 is valid. Suppose, for example, that $\{J_2, J_3, J_4\} = \{2, 3, 4\}$. Now suppose that emissions in the $n=2$ orbit reduce J_2 to $J_2 = 1$, so that the new values are $\{J_2, J_3, J_4\} = \{1, 3, 4\}$. Then from the electrical feasibility requirement given by (2.7) there must be emissions from $n=3$ to $n=2$, which results in $\{J_2, J_3, J_4\} = \{2, 2, 4\}$. This new solution is still not feasible because it again violates (2.7). Therefore it is concluded that rule R_3 is correct.

From this analysis it is clear that the assumption of R_1 leads to the solution for $\{J_n^*\}$, and therefore $\{Z_n^*\}$ is uniquely and correctly determined as given by (2.1). It is argued that that this rule stems from the principle of maximum feasible packing. As a counter example, suppose that $N_n = k^2 + i$, where k and i are integers and $k^2 + i < (k+1)^2$. Further suppose this value of N_n is feasible, in contradiction to Rule R_1 . Then let i keep increasing up to $k^2 + i = (k+1)^2$. During this entire move the new value of N_n is still feasible because the new value of S_n exceeds $2N_n$. Thus, Rule R_1 is valid.

Therefore, in conclusion, it has been shown that the Law of Noble Elements (LNE) is correct, so that the

only possible values of the atomic numbers of the noble elements is given by (2.1) as follows:

$$\{Z_n^*\} = \{2, 10, 18, 36, 54, 86, 118\}.$$

4. Proof of the Law of Radii (LOR)

LOR is given in (2.2) as $r_n = n^2 r_0$. It is noted that in the prior section this law was used to correctly derive the shell sizes of the periodic table. In addition, LOR is assumed and needed to derive the other experimentally verified results in Aucamp[1,2]. These applications represent a strong and convincing endorsement for LOR, so that it is argued the validity of this law is self-evident. In addition, the following analysis is added to further the case that LOR is correct.

Suppose another electron pair were included somewhere in the $\{Z_n^*\}$ vector already assumed in this work. As the entire set of atomic numbers is compact, then this would be equivalent to assuming the existence of a hitherto unknown element. There would then be two elements having the same atomic number. If it is postulated this cannot be the case, then this establishes LOR.

4. Conclusion

This work is the third and final paper in a series of three concerning atomic structure and radiation. In the first two papers (Aucamp[1,2]) the following two assumptions were made and not proved: (a) the Law of Radii (LOR) and (b) the Law of Noble Elements (LNE). While it can be argued that proofs of these laws are unnecessary because of the overwhelming theoretical and experimental evidence given in their support in all three of these papers, nevertheless in the interests of completeness proofs are covered herein.

5. References

1. Aucamp, D.C. (2023), "A Non-Radiating Atomic Electron Model with an Application to Molecular Structure and the Periodic Table", Open Access Journal of Physics, V5(1), 01-08.
2. Aucamp, D.C. (2023), "An Alternative to Quantum Theory, Photon Radiation, Entanglement, Photon Structure, the Wave-Particle Paradox, and Einstein's Photoelectric Effect", Open Access Journal of Physics, V5(1), 10-14.