

New Method for Solving the Problem of the Occurrence of

Solitons in an Open Water Channel

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ABSTRACT

On the basis of the certain form of write of an impulse equation the modeling of the solitary waves in the water channel is examined at action of gravitation forces. It is shown that as against an existing method of modelling where the waves propagating from left to right turn out from the equation of an impulse, and from right to left from the continuity equation in the offered technique both waves turn out from the equation of a solitary wave, its velocity and geometrical characteristics is submitted.

Keywords: The rectangular channel, a solitary wave, equation of impulse, KdV - equation.

INTRODUCTION

From times of the first supervision of solitary waves in water channel D.S. Russell in 1834, in the theory of the description of such waves has arisen set of various approaches [1, 2, 3]. For example, modelling of the solitary waves or solitons in [1] has allowed describe the waves propagating both from left to right, and from right to left. However, if the wave is propagated from left to right it is absolutely correctly that its modelling follows from the dynamic equation of an impulse but if the wave is propagated from right to left its modelling in [1] follows from the kinematic equation of continuity that causes questions. The matter is that elevation of a fluid in a solitary wave if not to take into account a surface tension it is defined by hydrostatic pressure in the basis of a solitary wave. In the equation of continuity there is no power parameter, therefore this equation from the physical point of view cannot model a solitary wave. In our opinion both opposite propagating

waves should appear proceeding from the equation of an impulse. It is possible if more correct to write down the initial equation of an impulse.

Researched solitary waves carry the name of solitons since their interaction is similar to interaction of particles. They can be reflected from solid border is similar to particles. At interaction with each other the solitons come apart keeping the structure constant. Such preservation of structure is defined by balance of nonlinear effects and dispersions on forward and back fronts of solitons.

SYSTEM OF THE HYDRODYNAMICS Equations

Let's solve a problem of occurrence of solitary waves in a nonviscous liquid at action of gravitational forces. The fluid flows in the channel with cross-section of the rectangular form, fig. 1.

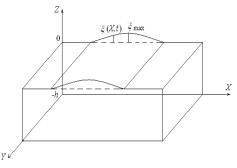


Figure1. Formation of a solitary wave (soliton) in the water channel

System of the hydrodynamics equations we shall write down as the equation of an impulse and the equation of continuity. Taking into account absence of the top solid border in the channel the equation of an impulse we shall write down as [4, 5, 6]:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial X} + W \frac{\partial W}{\partial X} + \frac{1}{\rho S} \frac{\partial PS}{\partial X} = 0, \qquad (1)$$

where *t* there is time, and streaming of a fluid in density ρ on axis *Y* is absent. We shall accept that the width of the channel along axis *Y* is equal to unit. The area of a stream in crosssection of channel is *S*. Pressure in fluid is *P*. Change of parameters of a stream is observed only on axes *X* and *Z*. Velocities of a fluid on these axes accordingly *V* and *W*.

It is examined no curl stream, so $\frac{\partial V}{\partial Z} = \frac{\partial W}{\partial X}$.

The equation of continuity it is used as:

$$\nabla^2 \varphi = 0, \qquad (2)$$

where φ there is the potential of velocity connected to components of velocity by the

formulas
$$V = \frac{\partial \varphi}{\partial X}$$
 and $W = \frac{\partial \varphi}{\partial Z}$.

At the further analysis we shall follow partly [1].

EXPANSION OF POTENTIAL IN SERIES

For the solution of system of the equations (1), (2) it is convenient to use dimensionless variables. For this purpose we shall enter scales of parameters of these equations:

$$X = M_X X^*; \qquad Z = M_Z Z^*; \qquad \xi = M_{\xi} \xi^*; \qquad P = M_P P^*;$$

$$\varphi = M_{\varphi} \varphi^*; \qquad t = M_t t^*; \qquad V = M_V V^*; \qquad W = M_W W^*.$$
(3)

where M_i there is scales of sizes, and dimensionless sizes are marked by asterisks. ξ is the current height of a solitary wave above unperturbed surface of water in the channel.

For write of scales concrete values and taking into account rather small value of a solitary wave amplitude ξ_{max} concerning depth of the channel *h* we shall enter small parameter $\varepsilon = \frac{\xi_{\text{max}}}{h} \ll 1$. Originally for us will be necessary only expressions for some scales. We shall accept:

$$M_{\xi} = \varepsilon h = \xi_{\max}, M_X = \frac{h}{\sqrt{\varepsilon}}, M_Z = h,$$

$$M_P = \varepsilon \rho g h, \qquad (4)$$

where *g* there is acceleration of free falling.

Let's transform the equation of continuity (2) to the dimensionless form.

Using
$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial X^2} + \frac{\partial^2 \varphi}{\partial Z^2}$$
, we shall write

down
$$\frac{\partial^2 \varphi^*}{\partial X^{*2}} \frac{M_{\varphi}}{\left(h/\sqrt{\varepsilon}\right)^2} + \frac{\partial^2 \varphi^*}{\partial Z^{*2}} \frac{M_{\varphi}}{h^2} = 0,$$

hence:

$$\mathcal{E}\frac{\partial^2 \varphi^*}{\partial X^{*2}} + \frac{\partial^2 \varphi^*}{\partial Z^{*2}} = 0.$$
 (5)

Let's shall expand the potential of velocity in series [1]:

$$\varphi^* = \sum_{n=0}^{\infty} \left(Z^* + 1 \right)^n \varphi_n^* \left(X^*, t^* \right), \tag{6}$$

where $\varphi_n^*(X^*, t^*)$ there is the dimensionless function dependent only from dimensionless longitudinal coordinate X^* and dimensionless time t^* .

Let's substitute the expansion in series (6) in the equation of continuity (5):

$$\sum_{n=0}^{\infty} \left(\varepsilon \left(Z^* + 1 \right)^n \frac{\partial^2 \varphi_n^*}{\partial X^{*2}} + n(n-1) \left(Z^* + 1 \right)^{n-2} \varphi_n^* \right) = 0 \quad (7)$$

We equate factors at $(Z^* + 1)^n$ to zero for what in the second addend in brackets it is replaced *n* on n+2. In result we shall find the recurrent formula:

$$\varphi_{n+2}^* = -\frac{\varepsilon}{(n+1)(n+2)} \frac{\partial^2 \varphi_n^*}{\partial X^{*2}}.$$
(8)

Thus, expansion in series (6) will be written down as:

$$\varphi^{*} = \varphi_{0}^{*} + (Z^{*} + 1)\varphi_{1}^{*} - \frac{\varepsilon}{2!}(Z^{*} + 1)^{2}\frac{\partial^{2}\varphi_{0}^{*}}{\partial X^{*2}} - \frac{\varepsilon}{3!}(Z^{*} + 1)^{3}\frac{\partial^{2}\varphi_{1}^{*}}{\partial X^{*2}} + \frac{\varepsilon^{2}}{4!}(Z^{*} + 1)^{4}\frac{\partial^{4}\varphi_{0}^{*}}{\partial X^{*4}} + \frac{\varepsilon^{2}}{5!}(Z^{*} + 1)^{5}\frac{\partial^{4}\varphi_{1}^{*}}{\partial X^{*4}} - \dots$$
(9)

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where φ_0^* there is dimensionless potential of a velocity at the bottom of the channel at $Z^* = -1$.

At the bottom of the channel at $Z^* = -1$ the velocity of a stream W = 0. Hence, proceeding

from (9) $\left(\frac{\partial \varphi^*}{\partial Z^*}\right) = \varphi_1^* = 0$. Therefore according

to the recurrent formula (8) $\varphi_{2n+1}^* = 0$. But $\varphi_n^*(X^*, t^*) \neq f(Z^*)$, therefore $\varphi_{2n+1} = 0$ at anyone Z^* , and not only at $Z^* = -1$.

Hence, expansion in series (9) can be copied as:

$$\varphi^* = \varphi_0^* - \frac{\varepsilon}{2!} \left(Z^* + 1 \right)^2 \frac{\partial^2 \varphi_0^*}{\partial X^{*2}} + \frac{\varepsilon^2}{4!} \left(Z^* + 1 \right)^4 \frac{\partial^4 \varphi_0^*}{\partial X^{*4}} - \dots$$
(10)

On a surface of water outside of a solitary wave $Z^* = 0$, hence:

$$\varphi^* = \varphi_0^* - \frac{\varepsilon}{2!} \frac{\partial^2 \varphi_0^*}{\partial X^{*2}} + \frac{\varepsilon^2}{4!} \frac{\partial^4 \varphi_0^*}{\partial X^{*4}} - \frac{\varepsilon^3}{6!} \frac{\partial^6 \varphi_0^*}{\partial X^{*6}} + \dots$$
(11)

Taking into account the small amplitude of the solitary wave the formula (11) it is possible to use and on a surface of solitary wave.

THE EQUATION OF AN IMPULSE

The equation of an impulse (1) can be transformed using the formula for cross-section of a fluid stream in the channel $S = h + \xi = h(1 + \varepsilon \xi^*)$ (the width of the channel is accepted to equal unit):

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial X} + W \frac{\partial W}{\partial X} + \frac{1}{\rho \left(1 + \varepsilon \xi^*\right)} \frac{\partial P \left(1 + \varepsilon \xi^*\right)}{\partial X} = 0.(12)$$

Passing in (12) to dimensionless variables according to (3), and packing scales we have:

$$\frac{\partial V^{*}}{\partial t^{*}} + \varepsilon V^{*} \frac{M_{\varphi}M_{t}}{M_{Z}^{2}} \frac{\partial V^{*}}{\partial X^{*}} + W^{*} \frac{M_{\varphi}M_{t}}{M_{Z}^{2}} \frac{\partial W^{*}}{M_{Z}\partial X^{*}} =$$
(13)
$$= -\frac{1}{\rho \left(1 + \varepsilon \xi^{*}\right)} \frac{M_{P}M_{t} \partial P^{*} \left(1 + \varepsilon \xi^{*}\right)}{M_{\varphi} \partial X^{*}}$$

Let's accept the ratio of scales which is not contradicting (4) as:

$$\frac{M_{\varphi}M_t}{M_Z^2} = \frac{M_PM_t}{\rho M_{\varphi}} = 1.$$
⁽¹⁴⁾

In result we shall receive the dimensionless equation of an impulse:

$$\frac{\partial V^*}{\partial t^*} + \varepsilon V^* \frac{\partial V^*}{\partial X^*} + W^* \frac{\partial W^*}{\partial X^*} = -\frac{1}{\left(1 + \varepsilon \xi^*\right)} \frac{\partial P^* \left(1 + \varepsilon \xi^*\right)}{\partial X^*}.$$
 (15)

Further in the equations we shall reject all small addends proportional ε^2 and higher degrees. Let's consider the sum:

$$\varepsilon V^{*2} + W^{*2} = \varepsilon \left(\frac{\partial \varphi^*}{\partial X^*}\right)^2 + \left(\frac{\partial \varphi^*}{\partial Z^*}\right)^2 = \varepsilon \left(\frac{\partial \varphi^*}{\partial X^*}\right)^2$$

as $\left(\frac{\partial \varphi^*}{\partial Z^*}\right)^2 \sim \varepsilon^2$.

Therefore the equation (15) can be copied as:

$$\left(1 + \varepsilon \xi^* \left(\frac{\partial V^*}{\partial t^*} + \varepsilon V^* \frac{\partial V^*}{\partial X^*}\right) + \frac{\partial P^*}{\partial X^*} + \varepsilon \frac{\partial P^* \xi^*}{\partial X^*} = O(\varepsilon^2) \quad (16)$$

We assume that solitary waves on a surface of water are defined only by gravitational forces. In this case pressure at a level of unperturbed fluid at Z = 0 equal $P = \rho g \xi$. In the dimensionless form this ratio looks like $P^* = \xi^*$. Substituting it in (16) we find:

$$\frac{\partial V^*}{\partial t^*} + \varepsilon V^* \frac{\partial V^*}{\partial X^*} + \varepsilon \xi^* \frac{\partial V^*}{\partial t^*} + \frac{\partial \xi^*}{\partial X^*} + \varepsilon \frac{\partial \xi^{*2}}{\partial X^*} = O(\varepsilon^2).$$
(17)

Further we use expansion in series (11) as:

$$\varphi^* = \varphi_0^* - \frac{\varepsilon}{2} \frac{\partial^2 \varphi_0^*}{\partial X^{*2}} + O(\varepsilon^2).$$
(18)

Let's pass in (18) to dimensionless longitudinal velocity at a level of the channel bottom $V_0 = \frac{\partial \varphi_0^*}{\partial X^*}$ (an asterisk in this case it is not

used):

$$V^* = \frac{\partial \varphi^*}{\partial X^*} = V_0 - \frac{\varepsilon}{2} \frac{\partial^2 V_0}{\partial X^{*2}} + O(\varepsilon^2).$$
(19)

Change of longitudinal velocity on height of a fluid in the channel is possible according to (10) find under the formula:

$$V^* = \frac{\partial \varphi^*}{\partial X^*} = V_0 - \frac{\varepsilon}{2} \left(Z^* + 1 \right)^2 \frac{\partial^2 V_0}{\partial X^{*2}} + O\left(\varepsilon^2\right)$$
(20)

Further in the equations it is used dimensionless velocity of a fluid at a level of the channel bottom V_0

Substituting (19) in (17) and grouping addends as the sum of the main and first order on small parameter ε , we shall find:

$$\left(\frac{\partial V_0}{\partial t^*} + \frac{\partial \xi^*}{\partial X^*}\right) + \varepsilon \left(\xi^* \frac{\partial V_0}{\partial t^*} + V_0 \frac{\partial V_0}{\partial X^*} + \frac{\partial \xi^{*2}}{\partial X^*} - \frac{1}{2} \frac{\partial^3 V_0}{\partial t^* \partial X^{*2}}\right) = O(\varepsilon^2).$$
(21)

The first term in the main order (21) is defined by velocity of movement of a liquid (its acceleration), the second term is defined by rapidity of change of the solitary wave fronts. In connection with the big distinction of velocities of a fluid movement and propagation of a solitary wave, in the main order for the analysis we shall enter so-called "slow time" $\tau = \frac{\varepsilon}{2}t^*$. Doing replacement in the main order (21 [1] $\frac{\partial}{\partial t^*} \rightarrow \frac{\partial}{\partial t^*} + \frac{\varepsilon}{2} \frac{\partial}{\partial \tau}, \text{ we shall write down (21)}$

as:

$$\left(\frac{\partial V_0}{\partial t^*} + \frac{\partial \xi^*}{\partial X^*}\right) + \varepsilon \left(\frac{1}{2} \frac{\partial V_0}{\partial \tau} + \xi^* \frac{\partial V_0}{\partial t^*} + V_0 \frac{\partial V_0}{\partial X^*} + \frac{\partial \xi^{*2}}{\partial X^*} - \frac{1}{2} \frac{\partial^3 V_0}{\partial t^* \partial X^{*2}}\right) = O(\varepsilon^2)$$
(22)

THE SOLUTION OF THE EQUATION OF AN **IMPULSE**

For the solution of the equation (22) we shall search entering new arguments:

$$r = X^* - t^*; \qquad l = X^* + t^*.$$
 (23)

Concerning these arguments for functions in the equation (22) we search with the help of introduction of auxiliary functions $f(r;\tau)$ and $g(l;\tau)$:

$$\begin{aligned} \boldsymbol{\xi}^* &= \boldsymbol{\beta} \big[f(\boldsymbol{r}; \tau) + g(\boldsymbol{l}; \tau) \big]; \\ V_0 &= \boldsymbol{\beta} \big[f(\boldsymbol{r}; \tau) - g(\boldsymbol{l}; \tau) \big], \end{aligned} \tag{24}$$

Where, constant factor β we shall determine later.

In new variables the main order (22) is identically equal to zero:

$$\frac{\partial V_0}{\partial t^*} + \frac{\partial \xi^*}{\partial X^*} = \beta \frac{\partial (f-g)}{\partial t^*} + \beta \frac{\partial (f+g)}{\partial X^*} = \beta \left(\frac{\partial f}{\partial t^*} + \frac{\partial f}{\partial X^*} - \frac{\partial g}{\partial t^*} + \frac{\partial g}{\partial X^*} \right) = \beta \left(\frac{\partial f}{\partial r} \frac{\partial r}{\partial t^*} + \frac{\partial f}{\partial r} \frac{\partial r}{\partial X^*} - \frac{\partial g}{\partial l} \frac{\partial l}{\partial t^*} + \frac{\partial g}{\partial l} \frac{\partial l}{\partial X^*} \right) = \beta \left(-\frac{\partial f}{\partial r} + \frac{\partial f}{\partial r} - \frac{\partial g}{\partial l} + \frac{\partial g}{\partial l} \right) = 0$$

$$(25)$$

Let's substitute (24) in the second bracket of the equation (22):

$$\frac{1}{2}\beta\frac{\partial f}{\partial \tau} - \frac{1}{2}\beta\frac{\partial g}{\partial \tau} - \beta^{2}(f+g)\frac{\partial f}{\partial r} - \beta^{2}(f+g)\frac{\partial g}{\partial l} + \beta^{2}(f-g)\frac{\partial f}{\partial r} - \beta^{2}(f-g)\frac{\partial g}{\partial l} + 2\beta^{2}(f+g)\frac{\partial f}{\partial r} + 2\beta^{2}(f+g)\frac{\partial g}{\partial l} + \frac{1}{2}\beta\frac{\partial^{3} f}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + 2\beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} f}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial r^{3}} + \frac{1}{2}\beta\frac{\partial^{3} g}{\partial l^{3}} - \beta^{2}(f+g)\frac{\partial g}{\partial r} + \frac{1}{2}\beta\frac{\partial g$$

Carrying out simple transformations we shall find:

$$\frac{1}{2}\beta\left(\frac{\partial f}{\partial\tau} - \frac{\partial g}{\partial\tau} + 4\beta f\frac{\partial f}{\partial r} + 4\beta g\frac{\partial g}{\partial l} + \frac{\partial^3 f}{\partial r^3} + \frac{\partial^3 g}{\partial l^3}\right).$$
 (27)

Substituting (27) in (22) and grouping addends we have:

$$\frac{1}{2}\beta\varepsilon\left(\frac{\partial f}{\partial\tau} + 4\beta f\frac{\partial f}{\partial r} + \frac{\partial^3 f}{\partial r^3} - \frac{\partial g}{\partial\tau} + 4\beta g\frac{\partial g}{\partial l} + \frac{\partial^3 g}{\partial l^3}\right) = O(\varepsilon^2)$$
(28)

The ratio (28) can be assumed to within small parameter ε the equation for functions f and g. That this equation represented the sum of two standard equations of Korteweg-de Vries [1], we 3

shall choose
$$\beta = \frac{3}{2}$$
:
 $\left(\frac{\partial f}{\partial f} + 6f\frac{\partial f}{\partial f} + \frac{\partial^3 f}{\partial g}\right) + \left(-\frac{\partial g}{\partial g} + 6g\frac{\partial g}{\partial g} + 6g\frac{\partial g}{\partial g}\right)$

$$\left(\frac{\partial f}{\partial \tau} + 6f\frac{\partial f}{\partial r} + \frac{\partial^3 f}{\partial r^3}\right) + \left(-\frac{\partial g}{\partial \tau} + 6g\frac{\partial g}{\partial l} + \frac{\partial^3 g}{\partial l^3}\right) = 0$$

$$(29)$$

The equation (29) describes two solitary waves of Korteweg-de Vries for auxiliary functions fand g. The first bracket describes a wave propagating from left to right the second bracket - a wave from right to left. Thus, these waves are distributed in opposite directions gradually leaving from each other. After a while these waves will cease to influence each other therefore they can be examined separately. We shall consider a wave moving from left to right thus a wave propagating from right to left, we believe absent g = 0. In this case the solution of the equation of an impulse (22) will look like $\xi^* = V_0 = \beta f(r;\tau)$ where the function $f(r;\tau)$ satisfies to the KdV-equation:

$$\frac{\partial f}{\partial \tau} + 6f \frac{\partial f}{\partial r} + \frac{\partial^3 f}{\partial r^3} = 0.$$
(30)

One-wave (one-soliton) solution of the equation (30) is well-known [1]:

$$f = \frac{2k^{*2}}{\operatorname{ch}^{2} \left(k^{*} \left(r - 4k^{*2} \tau - r_{0}\right)\right)} = \frac{2k^{*2}}{\operatorname{ch}^{2} \left[k^{*} \left(X^{*} - \left(1 + 2k^{*2}\right)t^{*} - r_{0}\right)\right]},$$
(31)

Taking into account that dimensionless velocity of a solitary wave:

$$c^* = \frac{d\omega^*}{dk^*} = 1 + 2\varepsilon k^{*2},$$
 (32)

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where ω^* there is dimensionless cyclic frequency of a wave. With the account: at $k^* = 0$ the size $\omega^* = 0$, integrating (32), we find a dispersion ratio for a wave as:

$$\omega^* = k^* + \frac{2}{3} \epsilon k^{*3}.$$
 (33)

For plotting of graph of a solitary wave under the formula (31) we shall pass to dimensional variables. For size of rise of a fluid level in a solitary wave above unperturbed surface, fig. 1, we shall write down:

$$\xi^* = \frac{\xi}{\xi_{\text{max}}} = \beta f(r;\tau) = \frac{3k^{*2}}{\operatorname{ch}^2 \left[k^* \left(X^* - c^* t^* - r_0 \right) \right]}.$$
 (34)

From the formula (34) follows $3k^{*2} = 1$ and $k^* = 1$

$$k^* = \frac{1}{\sqrt{3}}.$$

Dimensionless velocity of a wave can be found as:

$$c^* = 1 + 2\varepsilon k^{*2} = 1 + \frac{2}{3} \frac{\xi_{\text{max}}}{h}.$$
 (35)

For a finding of a time scale we shall receive from (14) scale of function of a potential:

$$M_{\varphi} = \sqrt{\frac{M_Z^2 M_P}{\rho}} = \sqrt{\frac{h^2 \varepsilon g h \rho}{\rho}} = h \sqrt{\xi_{\max} g} .$$
(36)

Hence, according to (14) the time scale is equal:

$$M_t = \frac{M_Z^2}{M_{\varphi}} = \frac{h}{\sqrt{g\xi_{\text{max}}}}$$
(37)

Using (35) scales $M_X = \frac{h}{\sqrt{\xi_{\text{max}}/h}}$ (4) and

(37) for the formula (34) we shall find expression:

$$\xi = \frac{\xi_{\max}}{\operatorname{ch}^2 \left(\frac{1}{\sqrt{3}} \left(\frac{X}{M_X} - \left(1 + \frac{2}{3} \frac{\xi_{\max}}{h} \right) \frac{t}{M_t} \right) - \delta \right)} = \frac{\xi_{\max}}{\operatorname{ch}^2 \left(\sqrt{\frac{\xi_{\max}}{3h^3}} \left(X - t \left(1 + \frac{2}{3} \frac{\xi_{\max}}{h} \right) \sqrt{gh} \right) - \delta \right)}$$
(38)

where a constant component of a phase $\delta = k^* r_0$.

Similarly we find velocity of a fluid in a solitary wave. Using the scale of longitudinal velocity

$$M_V = \frac{M_{\varphi}}{M_X} = \frac{\xi_{\max}\sqrt{gh}}{h} = V_{0\max}$$
 specifying

interrelation between the maximal rise of a fluid in a wave and the maximal velocity of a fluid in it at a level of the channel bottom, we shall find:

$$V_0 = \frac{V_{0 \max}}{\operatorname{ch}^2 \left(\sqrt{\frac{\xi_{\max}}{3h^3}} \left(X - t \left(1 + \frac{2}{3} \frac{\xi_{\max}}{h} \right) \sqrt{gh} \right) - \delta \right)}$$
(39)

The received formula for velocity of a fluid at a level of the channel bottom similar to the formula (38) of a fluid rises in a solitary wave. In the formula (39) dimensional velocities of a fluid are used. Thus, in formation of a solitary wave participates all layers of a fluid in the channel, and not only surface a layer.

On fig. 2 the graph of a fluid rise in the solitary wave plotted for the conditional moment of time t = 0 and for the following parameters of model:

$$h = 5m$$
, $\xi_{\text{max}} = 0.2m$, $g = 9.8m/s^2$ is shown.

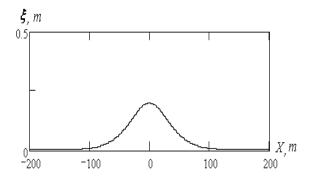


Figure 2. The graph of rise of a fluid in a solitary wave (soliton)

Thus the maximal velocity of a fluid in a solitary wave at the bottom of the channel

 $V_{0 \max} = \frac{\xi_{\max} \sqrt{gh}}{h} = 0.28 m/s$, and velocity

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of a solitary wave

$$c = \left(1 + \frac{2}{3} \frac{\xi_{\text{max}}}{h}\right) \sqrt{gh} = 7.93 \, m \, / \, s \, .$$

Therefore D.S. Russell was necessary to skip on a horse to overtake a solitary wave.

It is possible to find velocity of a fluid on a surface of the channel under the formula (19), and distribution of velocity of a fluid on height of the channel under the formula (20).

We shall note only that these velocities it is less than velocity of a fluid at a level of the channel bottom.

CONCLUSION

The form of the equation of an impulse of the nonviscous liquid is used allowing to solve a problem of a solitary wave modelling in the rectangular channel. The offered form of the equation of an impulse allows receive the solitary waves propagating in the channel in opposite directions only from the equation of an impulse. The equation of continuity at such modelling plays an auxiliary role and does not result in the equation of a wave. On the basis of the chosen parameters of a stream calculation of a fluid rise in a solitary wave, its velocity, and also velocity of a fluid at a level of the channel bottom are carried out. In formation of a solitary wave all layers of a fluid in the channel participate, and not only a surface layer.

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