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Fermions Confinement to a 4-Dimensional Universe is the Source for Dark Matter

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ABSTRACT

Fermions and photons, unlike gravitons, are confined to a 4-dimensional universe. Assuming a parallel universe similar to ours, allows, based on the confinement assumptions, to suggest a model that explains dark matter as a result of partially overlapping universes.

Keywords: *Fermions, Bosons, Confinement, Gravitons, Multiverse, Overlapping universes, Dark matter.*

Introduction

Our universe is made of two kinds of elementary particles. Fermions and Bosons
Fermions are massive half-spin particles.
Bosons are massless, integer spin force carrier entities.

Based on the principle of momentum conservation, it is speculated that at the initial moment of creation, several universes must have been created simultaneously, each with its own 4 dimensions. If matter is confined to 4 dimensions, it cannot pass from one universe to another and hence, parallel universes are transparent to the existence of each other, except for Bosonic (non-matter) interactions.

Based on this assumption, the existence of dark matter can be explained.

Kind of Matter in Universe

Fermions are one of the two fundamental classes of particles, the other being bosons. Fermion particles are described by Fermi–Dirac statistics and have quantum numbers described by the Pauli Exclusion Principle. They include quarks and leptons, as well as any composite particles consisting of an odd number of these, such as all baryons and many atoms and nuclei.

Fermions have half-integer spin; for all known elementary fermions this is 1⁄2. All known fermions, except neutrinos, are also Dirac fermions; that is, each known fermion has its own distinct antiparticle. It is not known whether the neutrino is a Dirac fermion or a Majorana fermion.

Fermions are the basic building blocks of all matter. They are classified according to whether they interact via the strong interaction or not.

In the Standard Model, there are 12 types of elementary fermions: six quarks and six leptons.

The Standard Model recognizes two types of elementary fermions: quarks and leptons. The model distinguishes 24 different fermions. There are six quarks (up, down, strange, charm, bottom and top quarks), and six leptons (electron, electron neutrino, muon, muon neutrino, tau particle and tau neutrino), along with the corresponding antiparticle of each of these.

Mathematically, fermions come in three types:

- Dirac fermions (massive),
- Weyl fermions (massless),
- Majorana fermions (each its own antiparticle).

Most Standard Model fermions are believed to be Dirac fermions, although it is unknown at this time whether the neutrinos are Dirac or Majorana fermions (or both). Dirac fermions can be treated as a combination of two Weyl fermions. However, Weyl Fermions are massless and hence lack internal coupling (unlike Dirac Fermions. Moreover, they may be considered as two massless particles moving in opposite directions. Confinement

Let us assume that there are two separate universes, coexisting, but undetectable to each other. In other words, in our 4-dimensional universe, we cannot observe anything in the parallel 4-dimensional universe.

Both universes make up an 8-dimensional universe, but it is made up of two completely independent of each other universes.

How can this be?

Suppose, all matter in our 4-dimensional universe is made up of Fermions, and massless photons, gravitons and other all force carrying entities.

Suppose now all but gravitons are confined to 4 dimensions.

This means that a parallel universe cannot be detected in our universe, since its force carrying entities can never reach our universe (they are confined in their own universe).

The only effect we may be able to detect, is the total mass of that universe. Gravitons from that universe can reach out to our universe.

This means, that when doing our gravitation calculations, there will be an extra matter present, which cannot be observed or explained as a result of solid matter.

We call this missing mass "Dark Matter".

DARK MATTER

Unlike normal matter, dark matter25 does not interact with the electromagnetic force, nor with any fermions. This means it does not absorb, reflect or emit light, making it impossible to spot. The existence of dark matter can only be inferred from the gravitational effect it seems to have on visible matter. Dark matter seems to outweigh visible matter roughly six to one. The matter we know and which makes up all stars and galaxies only accounts for 5% of the content of the universe. Dark matter is a form of matter that is thought to account for approximately 85% of the matter in the universe and about a quarter of its total energy density. The majority of dark matter is thought to be non-baryonic in nature, possibly being composed of some as-yet undiscovered subatomic particles. Its presence is implied in a variety of astrophysical observations 26, including gravitational effects that cannot be explained by accepted theories of gravity unless more matter is present than can be seen.

For this reason, most experts think dark matter to be abundant in the universe and to have had a strong influence on its structure and evolution. Dark matter is called dark because it does not appear to interact with observable electromagnetic radiation, such as light, and is thus invisible to the entire electromagnetic spectrum, making it extremely difficult to detect using usual astronomical equipment.

PHOTONS ARE CONFINED TO 4-DIMENSIONS

Photons dynamics are governed by 4dimensions equation of motion:

$$d\tau^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

With μ, ν=0, 1, 2, 3 and gμν the 4x4 gravitational 4-dimensions space metric.

If photons were to exist in more than 4 dimensions we would have:

$$d\tau^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

With $\alpha, \beta > 3$

And the speed of light c, (dx0=ct) will not be fixed for all reference frames.

Thus, photons must be confined in a 4-dimensional universe.

FERMIONS ARE CONFINED TO 4-DIMENSIONS

Based on group representation theory, Fermions are confined to 4-dimensions (see appendix).

The proof is based on Dirac equation (all Fermions are actually Dirac particles) and on the Dirac matrices (denoted here by the generalized matrices of n-dimensions instead of the conventional) anti commutation restriction (known as Clifford's constraint4):

$$\chi^\mu \chi^\nu + \chi^\nu \chi^\mu = 2g^{\mu\nu} I_N$$

This result was reached based on the following assumptions:

There exist, d Dirac gamma NxN matrices, where both d and N are integers.

These gammas are Lorentz covariant

The gammas must obey the Clifford algebra canonical anti-commutation relation

$$\{\chi^a, \chi^b\} = 2g^{ab} \quad \textit{With } a, b = (0{,}...d)$$

The number d specifies the total number of world coordinates, of which there are the usual x,y,z spatial coordinate, at least one is a temporal coordinate and the rest are extended coordinates (may be temporal and may be spatial). By convention, in our 4 dimensional world we use d=3+1, namely x,y,z and t).

Based on the calculus of the Dirac gamma matrices and their traces, it can be proven, that the existence of an irreducible group G, made of NxN matrices of size 2^d, is equivalent to the demand that $N^2 = 2^d$.

Adding pairs of gamma matrices over d=4, results in reducible representations. Hence d>4 is impossible.

The conclusion is that all matter particles (Dirac fermions) are confined in a 4-dimensional universe.

Gravitation is not Confined to 4-Dimensions

According to the general theory of relativity, gravitational attraction and influence is exerted by curvature of space. So far there is no evidence of any limitations on gravitation to be confined to 4-dimension. Space distortions can occur in one universe and across to other universes in other dimensions.

Gravitation and strings can be united if one assumes 11dimensions[11-16]. This stands in contrast to Fermions and photons being confined to 4-dimensions. It will therefore be assumed, that gravitation can pass freely between parallel universes.

One possible explanation of the need for 11 dimensions is the following:

4-dimensions are needed to describe an event in one universe. 3 dimensions are required to point to the position of the second universe with respect to the first one. Another set of 4 dimensions is required to describe the position of an event in the second universe. Thus 4+3+4 = 11

Parallel Universes and Higher Dimensions

In the quest for a unified theory, strings17-22 are candidates for unified quantum theory of gravity. They demand a higher dimensional universe (11 -26)8-11.

In spite of the fact that the universe is well described by a four-dimensional spacetime, there are several reasons why we consider theories in other dimensions. In some cases, by modeling spacetime in a different number of dimensions, a theory becomes more mathematically tractable, and one can perform calculations and gain general insights more easily.

One notable feature of string theories is that these theories require extra dimensions of spacetime for their mathematical consistency11. In bosonic string theory, spacetime is 26-dimensional12, while in superstring theory it is 10-dimensional13, and in M-theory it is 11-dimensional14-16.

There is a contradiction between the Fermionic confinement to 4-dimensions and the gravitation theories demanding 11 dimensions.

A Parallel Universe

According to big bang theory, and based on momentum conservation, there must be at least two universes created simultaneously and moving in opposite directions.

Suppose next, that in parallel to our universe, at the moment of creation, there were other universes created, parallel to ours. Those parallel universes may have the exact same content and size similar to our own universe, but with maybe a different particles/anti-particles ratio.

However, Fermions put a restriction – our material (fermionic) universe must be 4 dimensional. How can a parallel universe co-exist with ours?

A Fermion, cannot pass from one universe to a second universe because Fermions can only exist in 4 dimensions. This creates a confinement rule on Fermions. They are confined to that universe where they exist (created) in and cannot transit to other universes. Likewise, fermions from other universes can never cross into our material universe.

No massive particle in our universe can pass to another universe. However, photons and gravitons can pass freely between parallel universes.

Obviously, we will only have 4-dimensions in our fermionic universe, but there need to be more dimensions to include other parallel universes.

Since fermions can never exit their 4-dimensional universe, we say that fermions are confined.

The picture is thus the following:

We live in a 4-dimensional universe. Our universe is made of massive particles called Fermions. We can never exit our universe to other universes.

Suppose now, that Bosons, such as Photons and Gravitons, can pass freely from one universe to another. Thereof, information can be passed between these universes and so can forces of electromagnetism and gravitation.

But if a fermion exists in a parallel universe and confined in it, we will be able to observe it because light and gravitation can penetrate and transfer between the two parallel universes. This creates a contradiction because it means that fermions can exist in more than 4 dimensions.

The possible solution is the following.

Photons are confined just as fermions to 4 dimensions. This means that the fermions in the parallel universe can never be detected by us, but they can affect us by their mass (through gravitation).

So, we will assume now that fermions and photons are confined to 4 dimensions while gravitons are free to transfer between the two parallel universes.

This will explain "dark matter". A parallel universe which affects ours via gravitation but can never penetrate our universe and can never be observed by us. Only its mass affects our universe.

How can photons be confined to 4 dimensions?

If we look at Maxwell's equation, we can see its similarity to Dirac's equation. Same reasons for the confinement of Dirac's particles to 4 dimensions can be applied to Maxwell's particles.

Here is a summary of what we have:

- Fermions cannot exist in a universe of other than 4 dimensions.
- There may exist at least two parallel universes, each of 4 dimensions.
- Fermions may exist in each universe separately.
- Fermions are confined inside a universe and cannot transfer between universes.
- According to Maxwell's equation, and the constancy of the speeds of light, Photons can be considered to be confined in 4-dimensional.
- Hence, Fermions and Photons are confined in 4-dimensional universes.
- Gravitons are not confined.

Therefore, Fermions may be affected by gravitational forces from another universe, but cannot be seen outside their universe.

This may be the clue to "dark matter", where unexplained gravitational forces are predicted to exist, however no massive particles are observed, to justify these forces.

PARTIALLY OVERLAPPING UNIVERSES

If two parallel universes are of radius R each, and their centers offset from each other by an amount h, then the gravitational force exerted by one on the second will not be of full masses but rather, according to Newton's shell theorem, only partial attraction. This attraction will depend on the offset h:

- A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its center.
- If the body is a spherically symmetric shell (i.e., a hollow ball), no net gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.

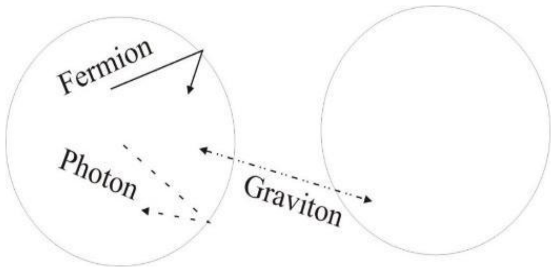

Figure1. *Fermions and Photons are confined. Gravitons are not confined*

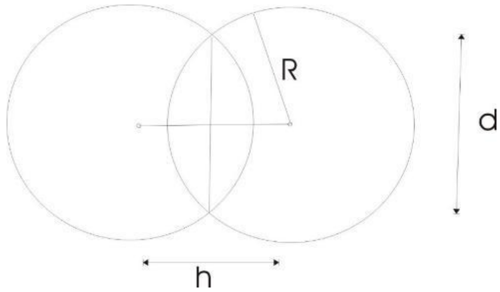

Figure2. *Overlapping universes or radius R and separation h*

The overlapping lens characteristics are determined by the universe radii R, by the separation of their centers h.

The width of the lens is designated by d.

The overlapping lens volume is given by:

$$V_{lense} = \frac{2\pi}{3}(R - h/2)^2(2R + h/2)$$

And compared to the universe volume V, the ratio is:

$$\frac{V_{lense}}{V} = \frac{(R - h/2)^2(2R + h/2)}{2R^3}$$

- If h = 0 (no offset) then there will be no gravitational effect at all.

- When h ≥ 2R the two universes become separate and the gravitational force between the two universes will start decreasing as 1/h3
- For 0 < h < 2R the gravitational force between the two universes will increase linearly.

The additional dark matter can be attributed to this separation offset, and to the ratio between lens volume to universe volume.

It is this lens material from the other universe that provides a possible explanation of the dark matter. The material from the other parallel universe outside the overlapping lens will attract our universe center as a whole. It is the overlapping part which affects our universe internally, and is the source for the gravitational deviations observations in spiral galaxies.

Conclusion

Based on Fermionic and Photonic confinement in a 4-dimensional universe, it is suggested, that only gravitation of one universe can affect other universes. Only overlapping parts of universes can have internal influences on one another. Non-overlapping parts will only have a mutual external attraction effect that will not be detectable.

That overlapping part may be the source for what is known as dark matter effects.

Moreover, based on the suggested theory, one may not be able to directly detect dark matter by photons interactions, irrespective of their energies, as photons cannot cross from one universe to another.

Appendix

Fermions Must be Confined to d=4

A d>4 Universe

Starting with the Dirac equation with N-dimensional spinors, we accept the fact that the gamma matrices, irrespective of the number of dimensions of our universe, they always must conform with the Clifford algebra restriction.

Following this, it is proven in this work, that there must exist a connection between the dimensionality of these matrices (and hence of the spinors), and the number of dimensions d of our universe.

Unlike previous approaches16, here we do not discriminate between odd and even dimensions. This leads us to the conclusion that N2=2d. Therefore, d must be even, and N must be an integer power of 2.

By a unique construction procedure, we use the gamma matrices to construct a relevant group of matrices. These groups presentations become reducible for any d > 4.

Thus, only d = 4 will be allowed, and our fermionic world cannot be other than a 3+1 universe.

N-Dimensional Representation of Dirac Equation

Any massive fermion must obey, by its definition, Dirac's equation

$$\left(i\hbar\gamma^{\mu}\,\partial_{\mu} - mcI_{N}\right)\Psi = 0$$

This is a must because of Energy Momentum considerations for a massive particle, and Lorentz covariance.

Dirac's matrices1 must obey

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu} \qquad (\mu = 0,1,2,3)$$

In a 3+1 world, gμν is a 4x4 matrix, so are 4x4 matrices. However, in an N-dimensional world, one needs NxN matrices4, with N >= 4, to set up a system with the properties required.

To extend to other dimensions, I will use the notation instead of these χ, NxN matrices, replace the 4x4 matrices in Dirac 4x4 equation. Thus one extends the Dirac equation to a NxN equation:

$$\left(i\hbar\chi^{\mu}\partial_{\mu} - mcI_{N}\right)\Psi = 0$$

Here is now a complex N-vector, IN is the NxN unit matrix, and are [**N x N**] matrices, satisfying the Clifford[2] requirement:

$$\chi^{\mu}\chi^{\nu} + \chi^{\nu}\chi^{\mu} = 2g^{\mu\nu}I_{N}$$

It is straight-forward by induction procedure, to go from a set of d-2 matrices in a a d-2 universe to a d dimensional universe by adding 2 gamma matrices from any given set of d-2 matrices. This is done by way of constructing

$$\chi^{\mu} = \begin{bmatrix} & \gamma_{\mu} \\ \gamma_{\mu} & \end{bmatrix}$$

So, from a set of d NxN matrices, one can have a set of d+2, [2N x 2N] matrices, satisfying the extended Clifford condition.

This procedure will thus create presentations of SO (N) to fermions in d-dimensional universe, for any even or odd d.

Group Representation and Odd Restriction on d

The fermionic matrices (because of convention, the 4x4 fermi matrices are called Dirac matrices or Gamma matrices. In here the terminology is mixed). According to Dirac's solution, these matrices must satisfy the canonical anti-commutation relation $\{\chi^{\mu},\chi^{\nu}\} = 2g^{\mu\nu}$. Other bases are possible, and are related to the chiral basis by rotations.

N-dimensional fermionic gamma matrices[4, 5, 13-16,] are a generalization of the four-dimensional Gamma matrices of Dirac to arbitrary dimension N. They are utilized in relativistic invariant wave equations for fermions (such as spinors) in arbitrary space-time dimensions, notably in string theory and super gravity[11, 14, 15].

Consider a d-dimensional space-time, with a flat Minkowski metric gab where a, b = 0, 1, ..., d−1. (The original Dirac matrices correspond to taking d = N = 4).

For a d-dimensional space (d-1 spatial + 1 temporal) dimension, there are d such matrices i (i =0, … d-1), of size N×N each, adhering the Clifford algebra $C\ell_{1,d-1}(\mathbb{R})$ anti commutator relation, $\{\chi_a,\chi_b\} = 2g_{ab}I_N$ Using the matrices IN and (a total of d+1 matrices) we can construct a set of 2^d N × N matrices as follows:

$I_N, \chi^{\mu}, \chi^{\mu}\chi^{\nu}, \chi^{\mu}\chi^{\nu}\chi^{\lambda}, \ldots \chi^0\chi^1\chi^2 \ldots \chi^{d-1}$

Over all combinations of indices, where $\mu < \nu < \lambda < \cdots$ etc

Notice that there is no restriction so far made of whether d is even or odd.

It can then b proved that

$$N^2 = 2^D$$

(See Appendix for details).

Hence, D must be an even number.

Is G an Irreducible Representation Group?

We create all the possible classes of G and look at their dimensions. Denoting the class of an element tg $\in \mathbb{G}$ we have:

$$[\![\Gamma_x]\!] = \{g\Gamma_x g^{-1} \ \ \forall g \in \mathbb{G}\}$$

For the group G it is easy to see that the conjugate classes are

$$\{I_N\},\{-I_N\},\{\chi^{\mu}\},\{-\chi^{\mu}\}\ldots\{\chi^0\chi^1\chi^2 \ldots \chi^{d-1}\}$$

Therefore, according to the Decomposition Theorem:

$$\sum_{\alpha} n_{\alpha}|\mathrm{Char}(\alpha)|^2 = |\mathbb{G}|$$

If and only if G is irreducible

Here Char (α) is the trace of the class, and n is the number of elements in the class.

As we have from Appendix I, all traces of the conjugate classes are null, except for $\mathrm{tr}(I_N) = N$ and $\mathrm{tr}(-I_N) = -N$.

For the group G to be irreducible one must have

$$\sum_{\alpha} n_{\alpha}|\mathrm{Char}(\alpha)|^2 = 1 \cdot N^2 + 1 \cdot (-N)^2 = 2N^2$$

In other words, for G to be irreducible, one must have

$$2N^2 = |\mathbb{G}|$$

But, as we saw above, if G is irreducible, then $|\mathbb{G}| = 2^{D+1}$

So, if $2N^2 = |\mathbb{G}|$ the group is irreducible, and if G is irreducible, then $|\mathbb{G}| = 2^{D+1}$

Therefore, the group is irreducible if, and only if, $N = 2^{D/2}$. Any other representation with N and D that do not satisfy the above, the representation will be reducible, or, if the relationship of N and D satisfy the above, then that representation must be irreducible.

The above result was obtained independent of D and it shows that D must be even.

It is therefore, that only even d dimensional universe is acceptable for fermions.

The representations are confined to those where N is even only (N= 4, 8, 16,… etc.).

Weyl Equation and the Neutrino

In the case of a massless fermion, m=0 and Dirac equation becomes

$$\chi^{\mu}\,\partial_{\mu}\Psi = 0$$

This is Weyl's equation. It has a solution given by N=2 matrices

$$\chi^{\mu} = (I_2, \sigma^x, \sigma^y, \sigma^z)$$

Where i are Pauli's 2x2 matrices, satisfying $\{\sigma^i, \sigma^j\} = 2\delta_{ij}\ldots$

One should not be misled here to take it as a 3-d representation, since the group will lose its

Lorentz covariance. Also, it will lack the time component. Thus, this set should be written as

$$\chi^{\mu} = (\sigma^0, \sigma^1, \sigma^2, \sigma^3)$$

With $\sigma^0 = I$ and $\sigma^1, \sigma^2, \sigma^3 = \sigma^x, \sigma^y, \sigma^z$

It has 4 matrices σ^{μ} satisfying $\{\sigma^{\mu}, \sigma^{\nu}\} = 2\delta^{\mu\nu}$ and provides a N=2 representation, in a d=4 universe, of a massless particle.

If neutrino has m=0, it cannot be a fermion since it will be a Weyl particle with d=4 N=2 which is impossible for a fermion under the requirement $N^2=2^d$. (N=2 requires d=2).

Otherwise, if m>0, it will fall under N=4, d=4 and can be a fermion of spin 1/2.

For an odd d (and an odd number of matrices), one should count the unit matrix I, as part of the group generators.

Therefore, a representation, based on $\sigma_x, \sigma_y, \sigma_z$ is not a proper representation of fermions.

By setting m=0 in

$$(\sigma_0\, \partial_t + \sigma_x\, \partial_x + \sigma_y\, \partial_y + \sigma_z\, \partial_z)\Psi = 0$$

$$\sigma^{\mu}\, \partial_{\mu} \Psi = 0$$

We see that by use of real components 2-vectors instead of a complex single 4-vector, there are two equations instead of one, and some coupling mechanism between the real part 2-vector and imaginary part 2-vector.

Notice that this coupling interaction does not vanish with mass (m→0). However, in the case of m=0, one cannot discern between the two 2-vectors R and I.

As we know, the Dirac particles must obey:

$$\frac{\partial^2 \psi_A}{c^2\, \partial t^2} - \nabla^2 \psi_A = -\left(\frac{mc}{\hbar}\right)^2 \psi_A$$

$$\frac{\partial^2 \psi_C}{c^2\, \partial t^2} - \nabla^2 \psi_C = -\left(\frac{mc}{\hbar}\right)^2 \psi_C$$

$$\frac{\partial^2 \psi_B}{c^2\, \partial t^2} + \nabla^2 \psi_B = +\left(\frac{mc}{\hbar}\right)^2 \psi_B$$

$$\frac{\partial^2 \psi_D}{c^2\, \partial t^2} + \nabla^2 \psi_D = +\left(\frac{mc}{\hbar}\right)^2 \psi_D$$

And define the group

$$\mathbb{K}_D = G_D \cup (-G_D) = \{I_N, -I_N, \gamma^0, -\gamma^0, \gamma^1, -\gamma^1, \gamma^2, -\gamma^2, \ldots \gamma^{(D-1)}, -\gamma^{(D-1)} \ldots .\ \gamma^0\gamma^1\gamma^2 \ldots \gamma^D,.\ -\gamma^0\gamma^1\gamma^2 \ldots \gamma^D\}$$

For m=0 (Weyl Fermion) it reduces to two entities only ($\psi_A = \psi_C$ and $\psi_B = \psi_D$) satisfying:

$$\frac{\partial^2 \psi_A}{c^2\, \partial t^2} - \nabla^2 \psi_A = 0$$

$$\frac{\partial^2 \psi_B}{c^2\, \partial t^2} + \nabla^2 \psi_B = 0$$

Or, a single particle moving in two opposite directions

$$\psi = \psi_0 \cos(\vec{k} \cdot \vec{r} \pm \omega t)$$

Extending the Results to Bosons?

Many theories[8-15] of (N, d) universe make use of odd d. But as was shown here, odd-d universe is not allowed for fermions. The only bosons allowed to cross 4-dimensions are the gravitations, and any other form of gravitation interaction.

An Upper Limit on d

So far, it was shown that d must be an even number. But *is there an upper limit on d?*

For each dimension, there is a single NXN Dirac matrice γ.

Let us start with Clifford algebra generated by $N \times N$ γ^{μ}matrices, $\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$

With μ, ν = 0,1,2,⋯D and where the metric signature $g^{\mu\nu}$ = diag(+,-,-,-,…).

For different d-dimensions, construct the following sets of matrices:

Notice that from now on it is sufficient to concentrate on an even dimensional fermionic universe, as it was shown above.

For a D dimensional universe, the matrices, use I_N and γ^{μ} to construct the following set G_D of 2^D $N \times N$ matrices:

$$GD = \{I_N, \gamma^{\mu}, \gamma^{\mu}\gamma^{\nu}, \gamma^{\mu}\gamma^{\nu}\gamma^{\lambda}, \ldots \gamma^0\gamma^1\gamma^2 \ldots \gamma^D\}$$

Over all combinations of indices, where $\mu < \nu < \lambda$, ... etc.

Create next the set

-GD=

$$\{-I_N, -\gamma^{\mu}, -\gamma^{\mu}\gamma^{\nu}, -\gamma^{\mu}\gamma^{\nu}\gamma^{\lambda}, \ldots -\gamma^0\gamma^1\gamma^2 \ldots \gamma^D\}$$

Thus,

$\mathbb{K}_D = \{\Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \dots \Gamma_{2D-1}, \Gamma_{2D}, \dots, \Gamma_{2^{D+1}-1}\}$ With $|\mathbb{K}_D| = 2|G_D| = 2^{D+1}$

It is obvious that KD is a group of order 2D+1 and that KD contains a set of sub-groups:

$\mathbb{K}_1 \subseteq \mathbb{K}_2 \subseteq \mathbb{K}_3 \dots \subseteq \mathbb{K}_D$

Dimensions	Sets of Gamma matrices	Set Size
d=1	$S_1 = \{I, \gamma^0\}$	$\lvert S_1\rvert = 2^1$
d=2	$S_2 = \{I, \gamma^0, \gamma^1\}$	$\lvert S_2\rvert = 2^2$
d=3	$S_2 = \{I, \gamma^0, \gamma^1, \gamma^2\}$	$\lvert S_3\rvert = 2^3$
d=4	$S_3 = \{I, \gamma^0, \gamma^1, \gamma^2, \gamma^3\}$	$\lvert S\rvert = 2^4$
d=6	$S_6 = \{I, \gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^a, \gamma^b\}$	$\lvert S\rvert = 2^6$
d=D	$S_D = \{I, \gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^a, \gamma^b, \dots \gamma^{(D-1)}\}$	$\lvert S_D\rvert = 2^D$

All Γ_i (except for i=0,1) are or products of 's γ^μ's(μ =0,1,2,3). Hence, when two new matrices 4 and 5 are introduced in a 6-dimensional universe, we define A=45, and it is a straight forward procedure to show that i, A=0 for all i⊂ K4.

Hence, by Schur's Lemma, if A ($A \neq \lambda I$), commutes with all matrices $\Gamma_i \subset \mathbb{K}_4$of the group, then the representation is necessarily reducible.

Therefore, the group K6 must be a ***reducible*** representation.

This forces us to conclude, that no irreducible representation $\mathbb{K}_d$ may be found for d>4 and therefore, the fermionic universe must have d=4.

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