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A New Gate to NUMEROV'S Method

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ABSTRACT

In this paper we will transform Numerov's method into a representation of matrix form to solve Schrödinger equation. The validity of the new method (Matrix Numerov's Method) is tested by applying it to calculate spectra of bottomonium. We compare our results with the experimental observed masses and theoretically predicted results. The obtained results are found to be in good agreement with the experimental results. This method can be straightforwardly generalized for other meson states and generally for the heavy mesons.

INTRODUCTION

In quantum mechanics, Schrödinger equation is a partial differential equation that describes how the quantum state of some physical system changes with time. There is no other equation has been studied more profoundly in theoretical physics than the Schrödinger equation [1]. Many numerical methods e.g., matrix method [2], Numerov's method [3, 4] and eigenfunctions expansion method [5] have been used in solving SE.In this work, we will introduce a highly accurate method to solve Schrödinger equation by using the simplest possible method.Here we will first discuss solution of the time-independent 1-D Schrödinger equation [6] which is a problem almost identical to solve the radial wave in three dimensions. We will derive and use Numerov's method, which is a specialized integration formula for numerically integrating differential equations to transform it into a new representation of matrix form on a discrete lattice dependsonly on the displacement of grid d and the number of grid N by studying the stability of N and r_{max} fm, where $d = \frac{r_{max}}{N}$.The theoretical results are compared withpublished theoretical data [7], and recently published experimental data [8].

THE POTENTIAL MODEL OF BOTTOMONIUM MESONS

The potential model used in solving SEhas following form[9]:

$$V_N(r) = \frac{l(l+1)}{2\mu r^2} - \frac{4\alpha_s}{3} + br + \frac{32\pi\alpha_s}{9{m_b}^2}\delta(r)S_b S_{\bar{b}} \quad (1)$$

Where $S_b.S_{\bar{b}} = \frac{s(s+1)}{2} - \frac{3}{4}$, μ is the reduced mass of the quark and anti-quark, m_b is the mass of the bottom quark, and S is the total spin quantum number of the meson. For the $b\bar{b}$mesons, the parameters α_s, b, σ, and m_b are taken to be 0.3996, 0.1607GeV2, 1.7494 GeV and 4.8038 GeV respectivelyas in ref. [7].

TRANSFORMING NUMEROV'S METHOD INTO A MATRIX FORM

Numerov's Method is a specialized integration formula for numerical integration of the differential equation:

$$\psi''(x) = f(x)\psi(x) \quad (2)$$

For the time-independent 1D Schrödinger equation, we have:

$$f(x) = -2m\big(E - V(x)\big)/\hbar^2 \quad (3)$$

Then Numerov's method will take the next formula

$$\frac{-\hbar^2}{2m}\frac{(\psi_{i-1} - 2\psi_i + \psi_{i+1})}{d^2} + \frac{(V_{i-1}\psi_{i-1} + 10V_i\psi_i + V_{i+1}\psi_{i+1})}{12} = E\frac{(\psi_{i+1} + 10\psi_i + V_{i+1}\psi_{i\_1})}{12} \quad (4)$$

Now we will transform the well-known Numerov's method into a representation of matrix formon a discrete lattice depending only on the grid number *d* and the matrix size

N.Hence, Eq. (4) could be transformed into a matrix form as follows

$$\frac{-\hbar^2}{2m} A_{N,N}\, B^{-1}_{N,N} \Psi_i + V_N \Psi_i = E_i \Psi_i \quad (5)$$

For the 3D radial Schrödinger equation, by considering the natural units ħ =1 then,

$$\frac{-1}{2m} A_{N,N}\, B^{-1}_{N,N} \Psi_i + [V_N(r) + \frac{l(l+1)}{r^2}]\Psi_i = E_i \Psi_i \quad (6)$$

The previous equation represents our novel approach to describe Numerov's method.

Results And Discussion

A non-relativistic potential model is used to study heavy meson spectra by using the Matrix numerov's method. To check out the new method, we have firstly started by checking the stability by changing the value of N. Then, the theoretical spectra of 1S, 2S, 3S, 4S bottomonium states are extracted we set the order of the matrix N at r_{max} = 20 fm, it is obvious that the results are stable when N≥98 as shown in Fig (1).Secondly, the stability of the method is checked out by using different values ofr_{max} fm. The calculation of the theoretical spectrum of 1S, 2S, 3S and 4S bottomonium states and the distance between the quark-antiquarkr_{max} at N = 200, it is obvious that the results are stable when $r_{max} \geq 9$ fm as shown In Fig (2). According to the figures, the value of N ≥ **200** and the value of r_{max} ≥ **20**fm could be used to give the spectra of bottomonium that consist with the experimental data. The comparison between the experiments and theoretical spectra of other group to the matrix Numerov's calculations for some of the spectra of bottomonium is given in Table (1). Moreover, the radial wave functions obtained by using the new method which is the most complete description that can be given to a physical system and it is shown in Figure (3) and Figure (4).

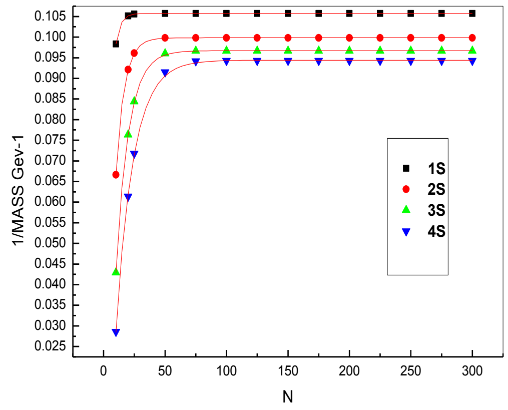

Figure1.*Shows the relation between the inverse of the theoretical spectrum of 1S, 2S, 3S and 4S bottomoniumstates and the order of the matrix N atr_{max} =20fm.*

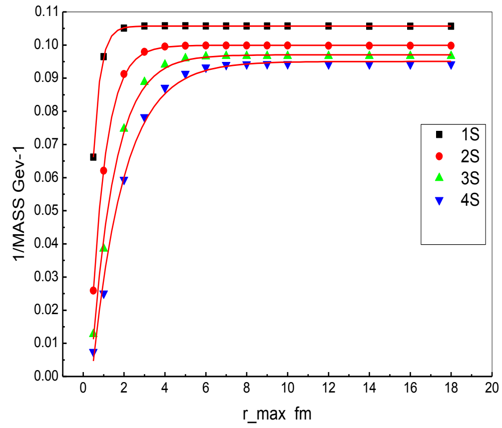

Figure 2.*The theoretical spectrum of 1S, 2S, 3S and 4S bottomonium states versus the distance between the quark- anti quark r_{max} at N = 200.*

Table1.*The obtained theoretical spectra of bottomonium$b\bar{b}$ compared to other groups*

State	Name	Theoretical masses [our work]in Gev	Theoretical masses of Aly in Gev [11]	Experimental masses in Gev [12]
1^1S_0	η_b(1S)	9.393	9.389	9,390.9±2.8
2^1S_0	η_b(2S)	9.996	9.994	
3^1S_0		10.33	10.328	
4^1S_0		10.596	10.593	
1^3S_1	Υ(1S)	9.458	9.459	9,460.30±0.26
2^3S_1	Υ(2S)	10.017	10.015	10,023.26±0.31
3^3S_1	Υ(3S)	10.345	10.354	10,355.2±0.5
4^3S_1	Υ(4S)	10.607	10.738	10,579.4±1.2
1^3P_2	χ_{b2} (1P)	9.936	9.935	9,912.21±0.40
2^3P_2	χ_{b2} (2P)	10.272	10.27	10,268.65±0.55
3^3P_J	χ_b (3P)	10.539	10.538	10,530±5
1^3P_1	χ_{b1} (1P)	9.904	9.912	9,892.76±0.40

2^3P_1	χ_{b1} (2P)	10.244	10.251	10,255.46±0.55
3^3P_J	χ_b (3P)	10.514	10.52	10,530±5
1^3P_0	χ_{b0} (1P)	9.884	9.879	9,859.44±0.52
2^3P_0	χ_{b0} (2P)	10.234	10.228	10,232.5±0.6
3^3P_J	χ_b (3P)	10.508	10.502	10,530±5
1^1P_1	h_b (1P)	9.92	9.92	
2^1P_1	h_b (2P)	10.258	10.258	
3^3P_J	χ_b (3P)	10.527	10.526	10,530±5

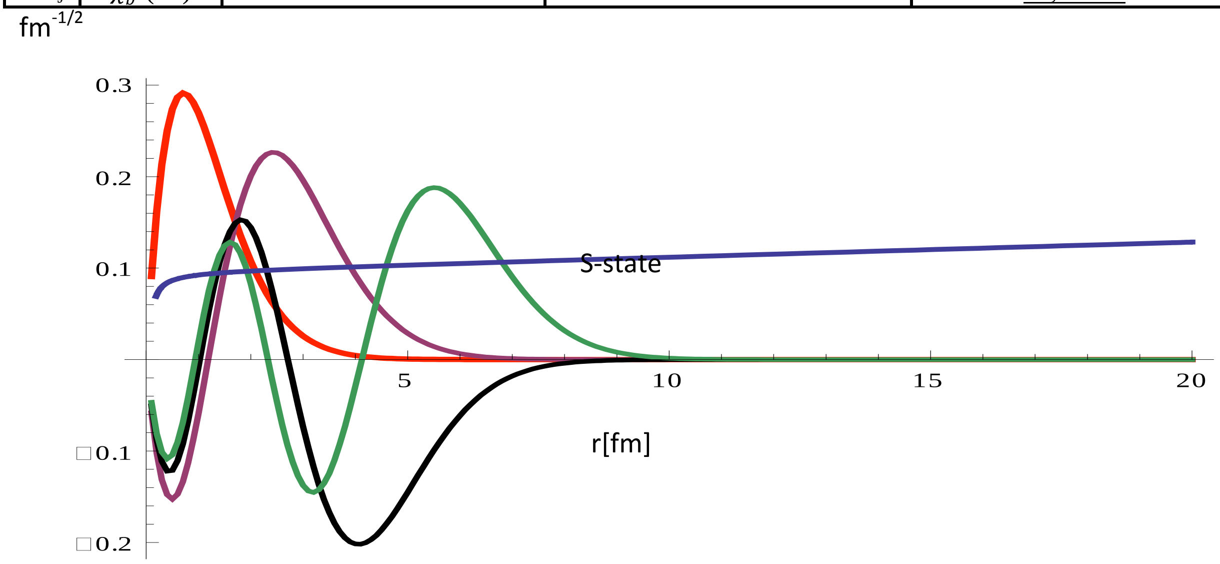

Figure 3. *bottomonium s-states reduced radial wave functions plotted together with used potential by using the new method*

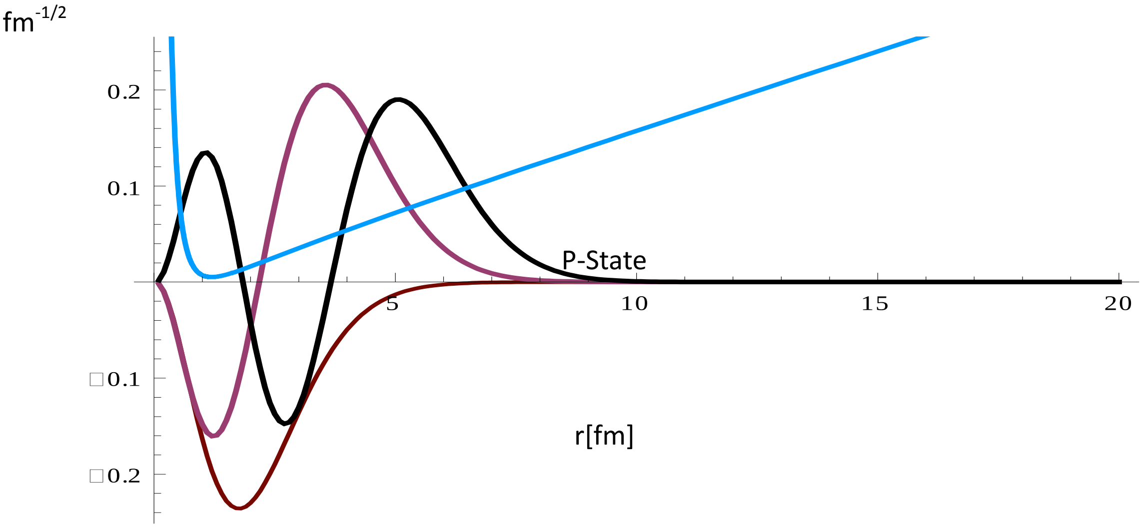

Figure 4.*Bottomonium P-states reduced radial wave functions plotted together with used POTENTIAL BY using the new method*

CONCLUSION

It is suggested that the Numerov's method is a reasonable method for solving Schrödinger's equation, so we reintroduced it by transforming it into a matrix form to solve radial Schrödinger's equation. As a remarkable result, we can point out that it is suggested to use the Matrix Numerov's method to solve 3D radial Schrödinger equation, for the following reasons:

- It is easier to use compared to other methods.
- Saving time, both in implementation and extraction results.
- Its accuracy in the theoretical calculations, it indicates a good agreement with the published experimental ones. Then we advise theoretical groups all over the world to check our new method by using several

types of potential models and apply it to study different cases of heavy mesons.

REFERENCES

[1] T. E. Simos,G. Mousadis. Computers Math. Applic**29**(1995)1

[2] J. P. Cooney,E. P. Kanter, Z. Vager, Am J Phys **49**(1981)76.

[3] L. Aceto , P. Ghelardoni, C. Magherini.Applied Numerical Mathematics **59** (2009)1644.

[4] M. Pillai, J.Goglio, and T. G. Walker. Am. J. Phys**80**(2012)1017.

[5] E.L. Koo, S. M. Cortes, G. V. Torres. Int J Qantum Chem, **56**(1995).

[6] P. Pedram, M. Mirzaei, and S. S. Gousheh. Journal of Theoretical and Applied Physics **7** (2013)34.

[7] A.A.Aly, M.SC Thesis, South Valley University, (2012).

[8] J. Beringer et al. (Particle Data Group,partial updating 2013 for the 2014 edition). Phys. Rev. **D86** (2012)010001.

[9] T.Barnes,S.Godfreyand,E.S.Swanson.Phys.Rev **D72** (2005)054026.

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