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Superconductivity Due to Condensation of Embedded Monopoles

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ABSTRACT

Exploring the role of quark monopoles (i.e. embedded monopoles) in restoration of chiral symmetry in SU(2) and SU(3) gauge theories, It has been demonstrated that these quark-monopoles (embedded monopoles) are tightly related to the chiral symmetry restoration and the resulting superconductivity in QCD. The SU(3) embedded monopoles have been classified into two categories according to maximal and minimal symmetry breakings respectively and it has been shown that in the case of QCD we can realize both these patterns of symmetry breaking simultaneously. It has also been shown that due to the role of embedded monopoles in chiral symmetry restoration and the fact that the chiral symmetry and confinement phenomenon are intimately related in QCD, the embedded monopoles play an important role in confining properties including superconductivity (rather color superconductivity).

Keywords: *Embedded monopoles; Chiral symmetry; Gauge theories; Confinement; Condensation; Super symmetry.*

INTRODUCTION

The monopoles [1-3] and dyons [4-6] became intrinsic part of all current grand unified theories[7] (GUT's) and super- symmetrical models[8-13]. Perhaps the most important aspect of monopoles and dyons in physics is their role in the mechanism of quark confinement[14-19] along the lines of dual Meissner effects[20-24] leading to dual superconductivity as discussed in our recent papers [25-36] by employing dual gauge potential where magnetic degree of freedom manifestly appears in the partition function. Embedded monopoles are gauge-invariant composite objects made of quark and gluon fields. These monopoles constitute a new class of defects of quantum chromo-dynamics (QCD) and proliferate in the quark-gluon plasma phase. This proliferation is associated with the well defined boundary [37] known as Kretesz-line, which separates the hadronic phase (i.e. the confinement phase) and the quark-gluon phase (i.e. de confinement phase) of QCD with realistic quark masses and vanishing chemical potential. At larger chemical potential, the phase transition re-emerges at a critical point and then continues as the first-order phase transition. At even higher temperature, more exotic phases such as color superconductivity and the color-flavor locking appear [38]. Embedded monopoles in QCD are analogues of the embedded Nambu-monopoles [39,40] in standard Electro-Weak model.[40]. There should be an indirect relation between embedded monopoles and confining properties including superconductivity since the confinement phenomenon and the chiral symmetry are intimately related in QCD and the embedded monopoles are considered as agents of chiral symmetry restoration [41].

Extending the restricted chromo dynamics (RCD) [31,32] in SU(2) and SU(3) gauge theories in the present paper by including quarks and gluons ,the study of dyonic condensation, quark confinement and superconductivity (dual superconductivity as well as color superconductivity) has been undertaken in extended RCD. In this paper the study of superconductivity due to embedded monopoles [37,41] in SU(2) and SU(3) gauge theories has also been carried out by exploring the role of quark monopoles (i.e. embedded monopoles) in restoration of chiral symmetry and the related confining properties. The bilinear functions of fermion fields have been constructed in SU (2) theory as scalar and axial vector from the point of view of space-time transformations. The corresponding Georgi-Glassow multiples have been used to construct gauge-invariant t' Hooft tensors in color space and the currents of quark-monopoles of three types have been shown to possess δ –singularities at corresponding world-lines. It has been shown that these quark-monopoles (embedded monopoles) are tightly related to the chiral symmetry restoration and the resulting superconductivity in QCD.

The bilinear functions of fermion field have also been constructed as octet vectors (scalar octet and axial octet) and the invariant octet vector has been shown transforming in the adjoint representation of the underlying symmetry group. Introducing three additional fields, the components of symmetric tensor field of rank-2 have been constituted and their transformations under axial rotations have been derived. These six octet fields have been used to describe the embedded monopoles in the SU(3) gauge theory. The magnetic charge of SU(3) embedded monopole has been constructed in terms of dual simple roots and its quantization condition in SU(3) theory has been derived. These SU(3) embedded monopoles have been classified into two categories according to maximal and minimal symmetry breakings respectively and it has been shown that in the case of QCD we can realize both these patterns of symmetry breaking simultaneously. It has also been shown that due to the role of embedded monopoles in chiral symmetry restoration and the fact that the chiral symmetry and confinement phenomenon are intimately related in QCD, the embedded monopoles play an important role in confining properties including superconductivity (rather color superconductivity).

Superconductivity Due To Condensation Of Embedded Monopoles In Su (2) Theory

Monopole condensation mechanism of confinement, together with dual superconductivity, implies that long range physics is dominated by Abelian degrees of freedom and the method of Abelian projection (i.e. Abelianization) is one of the popular approaches to the problem of confinement, and hence superconductivity, in non-Abelian gauge theories. In SU(2) gauge theory of QCD this Abeliazation may be achieved by the constraint given by [25-27].

$$D_\mu \hat{m} = \partial_\mu \hat{m} + ig\vec{V}_\mu \times \hat{m} = 0 \tag{2.1}$$

where D_μ is covariant derivative for the gauge group,μ = 0,1,2,3,V_μ is the generalized gauge potential and g is magnetic charge on monopole. The vector sign and cross product in this equation are taken in internal group space and $\hat{m}$characterizes the additional Killing symmetry (magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity i.e. $\hat{m}^2 = 1$

This magnetic symmetry imposes a strong constraint on the connection and hence may be regarded as symmetry of gauge potential. This gauge symmetry restricts not only the metric but also the gauge potential. Such a restricted theory (RCD) may be extracted from full QCD on restricting the dynamical degrees of freedom of theory by imposing magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential $V_\mu(A_\mu, B_\mu)$ of monopole to satisfy a strong constraint given by eqn.(2.1) which gives the following form of the generalized restricted potentials,

$$\vec{B}_\mu = A^*_\mu \hat{m} - \frac{1}{g}\hat{m} \times \partial_\mu \hat{m} \quad \text{with} \quad \vec{A}_\mu = B^*_\mu \hat{m} \tag{2.2}$$

whereA_μ and B_μ are the electric and magnetic constituents of gauge potential. These equations give as unrestricted Abelian components of the restricted potentials.

$$\hat{m}.\hat{A}_\mu = B^*_\mu \quad \text{and} \quad \hat{m}.\hat{B}_\mu = A^*_\mu \tag{2.3}$$

If $\vec{A}_\mu$in the original QCD then unrestricted potential is only B^*_μ and the restricted part of the potential is given as

$$\vec{B}_\mu = -\frac{1}{g}\vec{m} \times \partial_\mu \hat{m} = -\vec{W}_\mu \tag{2.4}$$

where W_μ is the potential of topological monopoles in magnetic symmetry which is entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. The unrestricted part of the gauge potential describes the monopole flux of color isocharges. The unrestricted part B^*_μ is the dual potential associated with charged gluons $W^{\pm}_\mu$ and leads to condensation of monopoles and the resultant state of chromo-magnetic superconductivity as shown in our earlier papers[25-30].

In the presence of a complex scalar field ϕ (Higg's field) and in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as

$$L = \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{2} | D_\mu \phi |^2 - V(\phi^* \phi) \tag{2.5}$$

where

$$V(\varphi^* \varphi) = -\eta(|\varphi|^2 - v^2)^2, \tag{2.6}$$

$$D_\mu \phi = (\partial_\mu + ig\, W_\mu)\phi, \tag{2.7}$$

and $H_{\mu\nu} = W_{\nu,\mu} - W_{\mu,\nu}$ (2.8)

In the presence of quarks (and gluons), the RCD Lagrangian (2.5) may be generalized to the following form;

$$L_R = \frac{1}{4} H^a_{\mu\nu} H^{\mu\nu}_a + \bar{\psi}^a (i\gamma^\mu D'_\mu)\psi_a + m\bar{\psi}^a \psi_a + \frac{1}{2}\left|D_\mu \phi\right|^2 - V(\phi^* \phi), \tag{2.9}$$

where a = 1, 2, 3, Ψ represents quark field with mass m, and $\vec{H}_{\mu\nu}$ has been constructed as $\vec{H}_{\mu\nu} = H_{\mu\nu}\vec{\xi}_3$ in the magnetic gauge by aligning $\hat{m}$ along a space- time independent direction (say $\hat{\xi}_3$ in isospin space) on imposing a gauge transformation U such that

$$\hat{m} \xrightarrow{U} \hat{\varepsilon}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.10}$$

with $H_{\mu\nu}$ defined by eqn. (2.8) where W_μ may be identified as the potential of topological dyons in magnetic symmetry which is entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of monopoles in magnetic gauge in which these objects appear as point-like Abelian ones and the gauge fields are expressible in terms of purely time-like non-singular physical potential W_μ. Lagrangian (2.9) can be used to represent the interactions between quarks and monopoles in the theory. It can be viewed as the effective Lagrangian used to describe the dual dynamics of RCD at the phenomenological level just as the Ginsburg – Landau Lagrangian is used in the theory of superconductivity. With this Lagrangian in hand, we have two phases in our theory. The first one is the un-confinement phase (quark-gluon plasma phase), where magnetic symmetry is preserved and the second one is the confinement phase (hadronic phase), where magnetic symmetry is broken dynamically. These two phases are separated by the well-defined boundary[37], known as Kreteszline, with which there is associated the proliferation of embedded monopoles which are gauge-invariant composite objects made of quark and gluon fields. Hence these embedded monopoles are also called quark-monopoles[37].

In order to explore the role of quark monopoles (i.e., embedded monopoles) in restoration of chiral symmetry and the related confining properties and superconductivity, in SU(2) theory, let us start with the quark field (i.e. fermion field) Ψ, introduced through eqn. (2.9), which transforms in the fundamental representation of gauge group SU(2) in Yang-Mills theory. Then the bilinear functions of this fermion field may be defined as

$$\hat{S}^a = \bar{\psi}(x)\hat{\tau}^a \psi(x) \tag{2.11}$$

$$\hat{A}^a = \bar{\psi}(x)(i\gamma_5)\hat{\tau}^a \psi(x) \tag{2.12}$$

where $\hat{\tau}^a$ are the Pauli matrices and $\hat{S}$ and $\hat{A}$ (the real valued composite fields) are scalar and axial (i.e., pseudo scalar) fields from the point of view of space-time transformations. Both these fields transform as adjoint three- component quantities with respect to the action of the gauge group.

Let ψ(x) used in equation (2.11) and (2.12) be c-valued function as an eigen mode of massless Dirac operator $\hat{D}$,

$$\hat{D}\psi_\lambda(x) = \lambda \psi_\lambda(x) \tag{2.13}$$

where

$$\hat{D} = \gamma_\mu(\partial_\mu + \frac{1}{2} i\tau^a B^a_\mu) \tag{2.14}$$

with $B^a_\mu(x)$ as the gauge fields. Let us consider the axial transformations $U_A(1)$ defined by the global Abelian parameter α as

$$\psi \to \psi' = e^{i\alpha\gamma_5}\psi \quad \text{and} \quad \bar{\psi} \to \bar{\psi}' = \bar{\psi} e^{-i\alpha\gamma_5} \tag{2.15}$$

Under these transformations $U_A(1)$, the color vector $\hat{S}^a$ and $\hat{A}^a$ given by equation (2.11) and (2.12), transform as

$$\hat{S}^a \to \hat{S}'^a = \hat{S}^a \cos 2\alpha + \hat{A}^a \sin 2\alpha$$
$$\hat{A}^a \to \hat{A}'^a = -\hat{S}^a \sin 2\alpha + \hat{A}^a \cos 2\alpha \qquad (2.16)$$

Let us construct the following three unit color vectors in terms of adjoint fields $\hat{S}^a$ and $\hat{A}^a$;

$$\hat{n}_s = \frac{\vec{S}}{|\vec{S}|}; \hat{n}_A = \frac{\vec{A}}{|\vec{A}|}; \hat{n}_I = \frac{\vec{S}\times\vec{A}}{|\vec{S}\times\vec{A}|}, \qquad (2.17)$$

where the symbol→ denotes vector in color space and $|\vec{S}|$ and $|\vec{A}|$ are norms of color vectors $\vec{S} = \hat{S}$ and $\vec{A} = \hat{A}$

$$|\vec{S}| = \left\langle \hat{S}, \hat{S} \right\rangle^{1/2},$$

$$|\vec{A}| = \left\langle \hat{A}, \hat{A} \right\rangle^{1/2}$$

The last relation in equation (2.17) gives the normalized vector product (in color space) of the scalar and axial color vectors $\hat{S}$ and $\hat{A}$ respectively. Using equations (2.17) and (2.16), it may readily be shown that the unit vector $\hat{n}_I$ is invariant under the axial transformations (2.15) and (2.16). Unit vectors of equation (2.17) may be interpreted as the directions of the composite adjoint Higgs field. Then we get following three Georgi-Glashow multiplets in SU(2) gauge theory with Higgs fields;

$$(\hat{n}_s^a, B_\mu^a); (\hat{n}_A^a, B_\mu^a); (\hat{n}_I^a, B_\mu^a) \qquad (2.18)$$

These multiplets can be used to construct the gauge invariant, t Hooft tensors[2] in the following form in color space:

$$\Im_{\mu\nu}^s = \hat{n}_s[\vec{H}_{\mu\nu} - \frac{1}{g}(D_\mu \hat{n}_s)\times(D_\nu \hat{n}_s)];$$

$$\Im_{\mu\nu}^A = \hat{n}_A[\vec{H}_{\mu\nu} - \frac{1}{g}(D_\mu \hat{n}_A)\times(D_\nu \hat{n}_A)];$$

$$\Im_{\mu\nu}^I = \hat{n}_I[\vec{H}_{\mu\nu} - \frac{1}{g}(D_\mu \hat{n}_I)\times(D_\nu \hat{n}_I)]; \qquad (2.19)$$

where

$$\vec{H}_{\mu\nu} = \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu + g(\vec{B}_\mu \times \vec{B}_\nu) \qquad (2.20)$$

is the field strength of the gauge field $\vec{B}_\mu$ with magnetic charge g and

$$(D_\mu)_{ab} = \delta_{ab}\partial_\mu + g \in_{abc} B_\mu^c \qquad (2.21)$$

is the adjoint covariant derivative. t' Hooft tensors of equation (2.19) are the gauge invariant field strength tensors for the diagonal components

$$B_\mu^s = (\vec{B}_\mu, \hat{n}_s);$$
$$B_\mu^A = (\vec{B}_\mu, \hat{n}_A);$$
$$B_\mu^I = (\vec{B}_\mu, \hat{n}_I); \qquad (2.22)$$

of the gauge field with respect to the color directions. The current of the quark monopole of S^{th} type is then given by

$$k_\mu^s = \frac{g}{2k}\Im_{\mu\nu,\nu}^{(d)s}$$
$$= \int_{C^s} d\tau \frac{\partial X_\nu^{C^s}(\tau)}{\partial\tau}\delta^{(4)}[x - X^{C^s}(\tau)] \qquad (2.23)$$

where $\Im_{\mu\nu}^{(d)s}$, dual of $\Im_{\mu\nu}^s$, is given by

$$\Im_{\mu\nu}^{(d)s} = \frac{1}{2}\in_{\mu\nu\sigma\rho}\Im_{\sigma\rho}^s \qquad (2.24)$$

and C^s is the corresponding world-line while monopole world-line is parameterized by the vector $x_\mu = X_\mu^{C^s}(\tau)$ and the parameter τ. This current, given by equation (2.23), has a δ-like singularity at the world-line C^s. Similarly, the currents of quark monopoles of A^{th} and I^{th} type have δ-like singularities at the world lines C^A and C^I respectively.

The quark monopoles defined by equation (2.23) are quantized and the corresponding monopole charge is conserved. In other words the world lines C^S, C^A and C^I are closed. . In the corresponding unitary gauges

$$n_s^a = \delta^{a3}, n_A^a = \delta^{a3} and n_I^a = \delta^{a3}, \qquad (2.25)$$

the quark monopoles correspond to monopoles embedded into the diagonal components given by equations (2.22).

Superconductivity Associated with Embedded Monopoles in Su(3) Gauge Theory

The magnetic structure of restricted chromo dynamics in SU(3) gauge theory may be described by two internal Killing vectors which commute with each other and also with the gauge symmetry itself and are normalized to unity. These Killing vectors are a λ_3- like octet $\hat{m}$ and its symmetric product $\hat{m}'$ which is λ_8 -like. Let us restrict the dynamical degrees of freedom of the theory (while keeping the full gauge degrees of freedom intact) by imposing the extra magnetic symmetry which restricts the generalized non-Abelian gauge potential $\vec{V}_\mu$ to satisfy the constraints given by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + ig\vec{V}_\mu \times \hat{m} = 0 \quad \text{and} \qquad (3.1)$$

$$D_\mu \hat{m}' = \partial_\mu \hat{m}' + ig\vec{V}_\mu \times \hat{m}' = 0,$$

where D_μ is covariant derivative for the gauge group and g is magnetic charge of monopoles. In these magnetic structures the eqn.(2.4) for restricted gauge potential may be generalized in to the following form for monopoles in restricted SU(3) gauge theory;

$$\vec{B}_\mu = -\frac{1}{g}(\hat{m}\times\partial_\mu\hat{m}) - \frac{1}{g}(\hat{m}'\times\partial_\mu\hat{m}') = -W_\mu - W'_\mu \qquad (3.2)$$

In the magnetic gauge $\hat{m}$ and $\hat{m}'$ become the space-time independent $\hat{\xi}_3$ and $\hat{\xi}_8$ respectively, where

$$\hat{\xi}_3 = \begin{pmatrix} 0\\0\\1\\0\\0\\0\\0\\0 \end{pmatrix} \quad and\ \hat{\xi}_8 = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\1 \end{pmatrix} \qquad (3.3)$$

Then the gauge potential of equation (3.2) may be written as

$$\vec{B}_\mu = W_\mu \hat{\xi}_3 + W'_\mu \hat{\xi}_8 \qquad (3.4)$$

where W_μ and W'_μ may be identified as the potentials of topological monopoles in magnetic symmetry of SU(3) gauge theory. These are entirely fixed by $\hat{m}$ and $\hat{m}'$, respectively, up to Abelian gauge degrees of freedom. The generalized field strength can, then, be constructed as

$$\vec{G}_{\mu\nu} = H_{\mu\nu}\hat{\xi}_3 + H'_{\mu\nu}\hat{\xi}_8 \qquad (3.5)$$

Where $H_{\mu\nu}$ is given by eqn.(2.8) and $H'_{\mu\nu}$ is defined as

$$H'_{\mu\nu} = W'_{\nu,\mu} - W'_{\mu,\nu} \qquad (3.6)$$

Then eqn. (2.9) may be generalized to the following form for the effective Lagrangian for SU(3) RCD in the presence of quarks;

$$L^R = L^R_{free} + L^R_i + \frac{1}{2}|D_\mu\varphi|^2 + \frac{1}{2}|D'_\mu\varphi'|^2 - V(\varphi^*\varphi\ ,\varphi'^*\varphi') \qquad (3.7)$$

Where,

$$D_\mu\phi = (\partial_\mu + igW_\mu)\phi;$$

$$D'_\mu\phi' = (\partial_\mu + igW'_\mu)\phi';$$

$$L^R{}_{free} = \frac{1}{4}[H_{\mu\nu}H^{\mu\nu} + H'_{\mu\nu}H'^{\mu\nu} + H_{\mu\nu}H'^{*\mu\nu} + H'_{\mu\nu}H^{*\mu\nu}] \qquad (3.8)$$

is the free field Lagrangian and

$$L^R_t = \overline{\psi}_r(i\gamma^\mu)[\partial_\mu + \frac{g}{3}W_\mu + \frac{g}{2\sqrt{3}}W'_\mu]\psi_r + \overline{\psi}_b(i\gamma^\mu)[\partial_\mu - \frac{2g}{3}W_\mu + \frac{g}{2\sqrt{3}}W'_\mu]\psi_b$$

$$+\overline{\psi}_y(i\gamma^\mu)[\partial_\mu + \frac{g}{3}W_\mu - \frac{g}{3}W'_\mu]\psi_y + m(\overline{\psi}_r\psi_r + \overline{\psi}_b\psi_b + \overline{\psi}_y\psi_y) \qquad (3.9)$$

where ψ_r, ψ_b, ψ_y, representing the quark triplet, constitute quark field ψ in SU(3) theory. Equation (3.9) is gauge extension of the Lagrangian given by (2.9) in SU(2) gauge theory. It leads to the dynamic condensation, color confinement and the resulting dual superconductivity in SU(3) theory in the presence of two scalar modes and two vector modes as the consequence of the presence of two-magnetic vectors $\hat{m}$ and $\hat{m}'$. Here also the theory has two phases i.e. the un-confinement phase (quark-gluon plasma phase), where magnetic symmetry is preserved and the confinement phase (hadronic phase), where magnetic symmetry is broken dynamically. These two phases are separated by the well defined boundary [37], known as Kreteszline, with which there is associated, the proliferation of embedded monopoles which are gauge-invariant composite objects made of quark and gluon fields.

In SU(3) gauge theory the bilinear functions of equations (2.11) and (2.12) are generalized into the following composite octet vectors;

$$\hat{S}^a = \bar{\psi}(x)\hat{T}^a\psi(x) \tag{3.10}$$

and

$$\hat{A}^a = \bar{\psi}(x)(i\gamma_5)\hat{T}^a\psi(x) \tag{3.11}$$

where $\hat{S}^a$ is the scalar octet, $\hat{A}^a$ is the axial octet and the gauge group is generated by eight traceless matrices

$$T^a = \lambda^a / 2, a = 1,2..........8 \tag{3.12}$$

normalized as

$$t_r T^a T^b = \frac{1}{2}\delta^{ab},$$

where λ^a are Gellmann matrices and t_r denotes the trace. In SU(3) gauge group there are totally asymmetric structure constants f^{abc} and totally symmetric constant d^{abc} defined via relations

$$[T_a, T_b] = T^aT^b - T^b T^a = if^{abc} T_c \tag{3.13}$$

and $\{T^a, T^b\} = T^a T^b + T^bT^a = \frac{1}{3}\delta^{ab} + d^{abc} T_c$

In SU(3) gauge group the following invariant field may also be built up from the octets given by eqns.(3.10) and (3.11);

$$\hat{I}^a = f^{abc}\hat{S}^b\hat{A}^c \tag{3.14}$$

which transforms in the adjoint representation of the SU(3) group. This relation is invariant under the global axial rotations (2.16) extended to octet fields of equations (3.10) and (3.11) as follows:

$$\hat{S}^a \to \hat{S}'^a = R_{11}\hat{S}^a + R_{12}\hat{A}^a$$
$$\hat{A}^a \to \vec{A}'^a = R_{21}\hat{S}^a + R_{22}\hat{A}^a,$$

where $R_{11} = R_{22} = \cos 2\alpha$ (3.15)

and $R_{12} = -R_{21} = \sin 2\alpha$

Since SU(3) gauge group possesses the symmetric structure constant d^{abc} through relation (3.3), we can construct three additional octet fields

$$\hat{S}^a_s; \hat{A}^a_s = \hat{S}^a_A; \hat{A}^a_A \tag{3.16}$$

which form a symmetric tensor field of rank two with the distinct components given as

$$\Im^a_{ss} = d^{abc}\hat{S}^b_s\hat{S}^c_s;$$
$$\Im^a_{SA} = d^{abc}\hat{S}^b_s\hat{S}^c_A;$$
$$\Im^a_{AA} = d^{abc}\hat{S}^b_A\hat{S}^c_A; \tag{3.17}$$

Under the axial rotation with transformations (3.15), these components transform as

$$\Im^a_{SS} \to \Im'^a_{SS} = R_{SS}R_{SS}\Im^a_{SS} + R_{SS}R_{SA}\Im^a_{SA} + R_{SA}R_{SS}\Im^a_{AS} + R_{SA}R_{SA}\Im^a_{AA}$$
$$\Im^a_{SA} \to \Im'^a_{SA} = R_{SS}R_{AS}\Im^a_{SS} + R_{SA}R_{AS}\Im^a_{AS} + R_{SS}R_{AA}\Im^a_{SA} + R_{SA}R_{AA}\Im^a_{AA}$$
$$\Im^a_{AA} \to \Im'^a_{AA} = R_{AS}R_{AS}\Im^a_{SS} + R_{AS}R_{AA}\Im^a_{SA} + R_{AA}R_{AS}\Im^a_{AS} + R_{AA}R_{AA}\Im^a_{AA} \tag{3.18}$$

Where

$R_{SS} = R_{11} = \cos 2\alpha$, $R_{SA} = R_{12} = -R_{AS} = -R_{21} = \sin 2\alpha$

And $R_{AA} = R_{22} = \cos 2\alpha$ (3.18a)

Substituting these values in equations (3.18), we get

$$\Im^a_{SS} \to \Im'^a_{SS} = \cos^2 2\alpha\Im^a_{SS} + \sin 4\alpha\Im^a_{SA} + \sin^2 2\alpha\Im^a_{AA};$$
$$\Im^a_{SA} \to \Im'^a_{SA} = -\sin\ 2\alpha\cos 2\alpha\Im^a_{SS} + \cos 4\alpha\Im^a_{SA} + \sin 2\alpha\cos 2\alpha\Im^a_{AA};$$
$$\Im^a_{AA} \to \Im'^a_{AA} = -\sin^2 2\alpha\Im^a_{SS} + \cos^2 2\alpha\Im^a_{AA}; \tag{3.19}$$

Thus in the realistic case of three colors, we have six independent structures which are classified with respect to the global axial rotations (3.5), as the scalar I^a, vectors $\hat{S}_a$ and $\hat{A}_a$, and rank-2 symmetric tensor with components $\Im^a_{ss}, \Im^a_{SA}$ and $\Im^a_{AA}$. All these structures behave as octet fields with respect to SU(3) gauge transformations. From the kinematical point of view any of these six octet fields can equivalently be used to construct the embedded quark monopoles in the SU(3) gauge theory[37]. Let us use a generic notation ξ^a for any of these six composite fields or for a linear combination of them. It is obvious from eqn. (3.9) that Cartan subgroup of the SU(3) gauge group is generated by the following two diagonal generators.

$$T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \qquad (3.20)$$

Both these are traceless Hermitian operators and we can form a two-component vector $\hat{T}$ in terms of these generators as

$$\hat{T} = (T^3, T^8) \qquad (3.21)$$

At any point of the space-time, the octet field ξ^a can be gauge rotated to the Cartan sub- algebra

$$\phi(x) \to |\phi(x)|(\vec{t}, \hat{T}) \qquad (3.22)$$

where $(\vec{t}, \hat{T})$ is the scalar product in Cartan space and $\vec{t}$ is the unit two component vector in the direction of composite octet ξ in the Cartan space i.e.

$$|\vec{t}\,| = 1 \qquad (3.23)$$

The magnetic charge of SU(3) quark- monopole may be written in the following form in terms of dual simple roots $\vec{\eta}_1^*$ and $\vec{\eta}_2^*$[42-44];

$$\vec{g}_M = \frac{1}{g}(n_1\vec{\eta}_1^* + n_2\vec{\eta}_2^*) \qquad (3.24)$$

where the dual roots $\vec{\eta}_1^*$ and $\vec{\eta}_2^*$ are expressed in terms of original simple roots $\vec{\eta}_1$ and $\vec{\eta}_2$ of SU(3) group as

$$\eta_1^* = \frac{\vec{\eta}_1}{|\vec{\eta}_1|^2} \text{ and } \eta_2^* = \frac{\vec{\eta}_2}{|\vec{\eta}_2|^2} \qquad (3.25)$$

In the self -dual form the roots $\vec{\eta}_1$ and $\vec{\eta}_2$ may be written as

$$\vec{\eta}_1 = (1,0) \qquad (3.26)$$

and $\vec{\eta}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ Then we have $|\vec{\eta}_1| = 1$ and $|\vec{\eta}_2| = 1$ In this case equations (3.25) give

$$\vec{\eta}_1^* = \vec{\eta}_1 = (1,0) \qquad (3.27)$$

and $\vec{\eta}_2^* = \vec{\eta}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$

Then the generalization of Dirac quantization condition of the embedded monopole charge to the case of SU(3) theory is given by

$e^{4\pi i g(\vec{g}_M, \hat{T})} = 1$ which gives

$$(\vec{g}_M, \hat{T}) = \frac{n}{2g} \qquad (3.28)$$

where n is an integer. Substituting equations (3.21), (3.24) and (3.22) in to this condition, we find that n_1 and n_2 of equation (3.24) must be integers.

SU(3) quark monopole may be classified[37] into following two categories according to the direction of local Higgs field in the Cartan sub-algebra;

- If the vector $\vec{t}$ in equation (3.22) is not orthogonal to any simple root $\vec{\eta}_1$ and $\vec{\eta}_2$ given by equation (3.26), then the pattern of symmetry breaking is maximal i.e.

$$SU(3) \to U(1) \times U(1) \qquad (3.29)$$

Then embedded monopoles are described by two integer numbers entering in equation (3.24). In this case, embedded monopoles corresponding to the two U(1) directions may be constructed as t' Hooft-Polyakov type monopoles. These monopoles will couple to electric gluons in the theory and this mixed mode interaction is crucial for confinement and chromomagnetic superconductivity.

- If the asymptotic Higgs field is orthogonal either to simple root $\vec{\eta}_1$ or to the simple root $\vec{\eta}_2$, then the symmetry breaking is minimal i.e.

$$SU(3) \to U(2) \qquad (3.30)$$

Then the monopoles are characterized by one integer number which is $n_1$if

$$(\vec{\eta}_2,\vec{t}) = 0 \tag{3.31}$$

or $n_2$if ($\vec{\eta}_1,\vec{t}$) =0

The nature of symmetry breaking pattern (either maximal or minimal) corresponds to the type of the embedding of the SU(2) t' Hooft – Polyakove monopoles into SU(3) gauge group. The choice of either maximal symmetry breaking given by equation (3.29) or minimal symmetry breaking (3.30) depends on the details of Higgs field[45]. In the case of QCD we can realize both the patterns of symmetry breaking simultaneously.

Discussion

In equations (2.11) and (2.12), the bilinear functions of the fermion fields have been constructed to explore the role of quark monopoles (i.e., embedded monopoles) in restoration of chiral symmetry and the related confining properties and superconductivity in SU(2) gauge theory. These scalar and axial fields transform according to equations (2.16) under the axial transformations given by equations (2.15). The unit vectors of equations (2.17) may be interpreted as the direction of the composite Higgs field and consequently, equations (2.18) give three Georgi-Glassow multiplets in SU (2) gauge theory with Higgs field. These multiplets have been used to construct gauge invariant t' Hooft tensors in the form given by equation (2.19) in color space. These tensors are gauge invariant field strength tensors for diagonal components of the gauge field with respect to the color direction. The current of the quark monopole ofS^{th} type, given by equation (2.23), has a δ-like singularity at the world line C^S. In the similar manner the currents of quark monopoles of A^{th} and I^{th} type may be constructed and these currents also may be shown to have δ-like singularities on the world lines C^A and C^I respectively. The quark monopoles described by equations (2.23) are quantized and the world lines C^S, C^A and C^I are closed. Thus the quark monopoles of S^{th}, A^{th} and I^{th} types carry the magnetic charges with respect to scalar, axial and chiral invariant components of gauge fields given by equations (2.22). In the corresponding unitary gauges, defined by equations (2.24a), the quark monopoles correspond to monopoles embedded into the diagonal field components given by equations (2.22). In the gauges, where these diagonal components are regular, such monopoles are hedgehogs[41] in the composite quark-antiquark fields. The corresponding quark condensates are characterized by the typical hedgehog behavior $n_S^a \sim x^a$ etc. in the local transverse vicinity of monopoles. The existence of these monopoles and their condensate in QCD is a kinematical consequence of the existence of adjoint real valued fields of equations (2.11), (2.12) and (2.17). There is an infinite number of equivalent formulations of the embedded monopoles (i.e., quark monopoles) associated with triplet isovectors given by chiral rotation (2.16) of isovectors $\hat{S}$ and $\hat{A}$ with an arbitrary angle 2α. Because of the hedgehog behavior of embedded QCD monopoles in quark – antiquark condensates, these monopoles are rightly called 'quark monopoles'. These quark monopoles are tightly related to the chiral symmetry restoration and the resulting color superconductivity in RCD.

In SU(3) gauge theory the bilinear functions of fermion field, given by equations (3.10) and (3.11), are octet vectors (scalar octet and axial octet) and the invariant vector, constructed in the form given by equation (3.14), transforms in the adjoint representation of the underlying symmetry group SU(3). The relation (3.14) is invariant under the global axial transformation given by equations (3.15). Three additional octet fields, given by equation (3.16), form a symmetric tensor field of rank two with the components given by equations (3.17). These components transform as equations (3.18) and (3.19) under the axial rotation given by equation (3.15). Thus in the realistic case of three colors, we have six independent structures which are classified with respect to the axial global rotations (3.15) as the scalar I^a, vectors $\hat{S}^a$ and $\hat{A}^a$ and the rank-two symmetric tensor with components $\Im^a_{SS}, \Im^a_{SA}$ *and* $\Im^a_{AA}$. All these structures behave as octet fields with respect to SU(3) gauge transformations. From the kinematical point of view any of these six octet fields can be used to construct the embedded monopoles in the SU(3) theory. Any of these octet fields can be gauge rotated to Cartan sub algebra according to equation (3.22) where $\hat{T}$ is a two-component vector defined by equation (3.21) in terms of the generators (3.20) of Cartan subgroup.

Equation (3.24) gives the magnetic charge of SU (3) embedded monopole in terms of the dual simple roots given by equation (3.25) or equation (3.26) in self-dual form. Dirac quantization condition (3.28) requires that n_1 and n_2 of equation (3.24) for charge of embedded monopole must be integers. Maximal or minimal natures of symmetry breaking pattern, given by relations (3.29) and (3.30) respectively, correspond to the type of the embedding of SU(2) t' Hooft – Polyakov monopoles into SU(3) gauge group. In the case of QCD we can realize both these patterns of symmetry breaking simultaneously.

The density of the embedded monopoles is high in the charily invariant phase while it is relatively low in the charily broken phase[37] and in the cores of these monopoles the chiral invariance is broken. Thus the embedded QCD monopoles are the agents of the chiral symmetry restorations. Thus in real QCD with dynamical quarks, the breaking of chiral symmetry must explicitly be seen in the densities of the embedded monopoles. Due to this role of quark monopoles in chiral symmetry restoration and also the fact that the chiral symmetry and confinement phenomenon are intimately related in QCD, the quark monopoles play an important role in confining properties including superconductivity (rather color superconductivity). In QCD the role of Higgs condensate is played by chiral condensate, which makes the role of embedded QCD monopoles meaningful physically [46,47], particularly in connection with color superconductivity.

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