

## Seismo Diffusion Coefficient Fractal Dimension for Characterizing Shajara Reservoirs of the Permo-Carboniferous Shajara Formation, Saudi Arabia

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### ABSTRACT

The quality and assessment of a reservoir can be documented in details by the application of seismo diffusion coefficient. This research aims to calculate fractal dimension from the relationship among seismo diffusion coefficient, maximum seismo diffusion coefficient and wetting phase saturation and to approve it by the fractal dimension derived from the relationship among capillary pressure and wetting phase saturation. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, seismo diffusion coefficient, and maximum seismo diffusion coefficient and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between seismo diffusion coefficient and maximum seismo diffusion coefficient versus logarithm wetting phase saturation. The slope of the first procedure =  $3 - D_f$  (fractal dimension). The second procedure for obtaining the fractal dimension was determined by plotting the logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure =  $D_f - 3$ . On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units.

**Keywords:** Shajara Reservoirs; Shajara Formation; Seismo diffusion coefficient fractal dimension; Capillary pressure fractal dimension

### INTRODUCTION

Seismo electric effects related to electro kinetic potential, dielectric permittivity, pressure gradient, fluid viscosity, and electric conductivity was first reported by <sup>[1]</sup>. Capillary pressure follows the scaling law at low wetting phase saturation was reported by <sup>[2]</sup>. Seismo electric phenomenon by considering electro kinetic coupling coefficient as a function of effective charge density, permeability, fluid viscosity and electric conductivity was reported by <sup>[3]</sup>. The magnitude of seismo electric current depends on porosity, pore size, zeta potential of the pore surfaces, and elastic properties of the matrix was investigated by <sup>[4]</sup>. The tangent of the ratio of converted eclectic field to pressure is approximately in inverse proportion to permeability was studied by <sup>[5]</sup>. Permeability inversion from seismoelectric log at low frequency was studied by <sup>[6]</sup>. They reported that,

the tangent of the ratio among electric excitation intensity and pressure field is a function of porosity, fluid viscosity, frequency, tortuosity, fluid density and Dracy permeability. A decrease of seismo electric frequencies with increasing water content was reported by <sup>[7]</sup>. An increase of seismo electric transfer function with increasing water saturation was studied by <sup>[8]</sup>. An increase of dynamic seismo electric transfer function with decreasing fluid conductivity was described by <sup>[9]</sup>. The amplitude of seismo electric signal increases with increasing permeability which means that the seismo electric effects are directly related to the permeability and can be used to study the permeability of the reservoir was illustrated by <sup>[10]</sup>. Seismo electric coupling is frequency dependent and decreases expontially when frequency increases was demonstrated by <sup>[11]</sup>. An increase of permeability with increasing pressure head and bubble pressure fractal

dimension was reported by [12,13]. An increase of geometric relaxation time of induced polarization fractal dimension with permeability increasing and grain size was described by [14,15].

**MATERIAL AND METHODS**

Sandstone samples were collected from the surface type section of the Permo-Carboniferous Shajara Formation, latitude 26° 52' 17.4", longitude 43° 36' 18". (Figure1). Porosity was measured on collected samples using mercury intrusion Porosimetry and permeability was derived from capillary pressure data. The purpose of this paper is to obtain seismo diffusion coefficient fractal dimension and to confirm it by capillary pressure fractal dimension. The fractal dimension of the first procedure is determined from the positive slope of the plot of logarithm of the ratio of seismo diffusion coefficient to maximum seismo diffusion coefficient  $\log (SDC^{1/4}/SDC_{max}^{1/4})$  versus  $\log$  wetting phase saturation ( $\log Sw$ ). Whereas the fractal dimension of the second procedure is determined from the negative slope of the plot of logarithm of  $\log$  capillary pressure ( $\log Pc$ ) versus logarithm of wetting phase saturation ( $\log Sw$ ).

The seismo diffusion coefficient can be scaled as

$$Sw = \left[ \frac{SDC^{1/4}}{SDC_{max}^{1/4}} \right]^{[3-Df]} \tag{1}$$

Where  $Sw$  the water saturation,  $SDC$  the seismo diffusion coefficient in square meter / second,  $SDC_{max}$  the maximum seismo diffusion coefficient in square meter / second

Equation 1 can be proofed from

$$SDC = \left[ \frac{A}{t} \right] \tag{2}$$

Where  $SDC$  the seismo diffusion coefficient in square meter / second,  $A$  the area in square meter,  $t$  the time in second

The area can be scaled as

$$A = \left[ \frac{Q}{V} \right] \tag{3}$$

Where  $A$  the area in square meter,  $Q$  the flow rate in cubic meter / second, and  $V$  the velocity in meter / second

Insert equation 3 into equation 2

$$SDC = \left[ \frac{Q}{t * V} \right] \tag{4}$$

The flow rate can be scaled as

$$Q = \left[ \frac{3.14 * r^4 * \Delta p}{8 * \eta * L} \right] \tag{5}$$

Where  $Q$  the flow rate in cubic meter / second,  $r$  the pore radius in meter,  $\Delta p$  the differential pressure in pascal,  $\eta$  the fluid viscosity in pascal \* second, and  $L$  the capillary length in meter

Insert equation 5 into equation 4

$$SDC = \left[ \frac{3.14 * r^4 * \Delta p}{t * V * L} \right] \tag{6}$$

The maximum pore radius can be scaled as

$$SDC_{max} = \left[ \frac{3.14 * r_{max}^4 * \Delta p}{t * V * L} \right] \tag{7}$$

Divide equation 6 by equation 7

$$\left[ \frac{SDC}{SDC_{max}} \right] = \left[ \frac{\left[ \frac{3.14 * r^4 * \Delta p}{t * V * L} \right]}{\left[ \frac{3.14 * r_{max}^4 * \Delta p}{t * V * L} \right]} \right] \tag{8}$$

Equation 8 after simplification will become

$$\left[ \frac{SDC}{SDC_{max}} \right] = \left[ \frac{r^4}{r_{max}^4} \right] \tag{9}$$

Take the fourth root of equation 9

$$\sqrt[4]{\left[ \frac{SDC}{SDC_{max}} \right]} = \sqrt[4]{\left[ \frac{r^4}{r_{max}^4} \right]} \tag{10}$$

Equation 10 after simplification will become

$$\left[ \frac{SDC^{1/4}}{SDC_{max}^{1/4}} \right] = \left[ \frac{r}{r_{max}} \right] \tag{11}$$

Take the logarithm of equation 11

$$\log \left[ \frac{SDC^{1/4}}{SDC_{max}^{1/4}} \right] = \log \left[ \frac{r}{r_{max}} \right] \tag{12}$$

$$\text{But; } \log \left[ \frac{r}{r_{max}} \right] = \frac{\log Sw}{[3 - Df]} \tag{13}$$

Insert equation 13 into equation 12

$$\frac{\log Sw}{[3 - Df]} = \log \left[ \frac{SDC^{1/4}}{SDC_{max}^{1/4}} \right] \tag{14}$$

Equation 14 after log removal will become

$$Sw = \left[ \frac{SDC^{1/4}}{SDC_{max}^{1/4}} \right]^{[3-Df]} \tag{15}$$

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Equation the 15 proof of equation 1 which relates the water saturation, seismo diffusion coefficient, maximum seismo diffusion coefficient, and the fractal dimension.

The capillary pressure can be scaled as

$$Sw = [Df - 3] * Pc * \text{constant} \quad 16$$

Where Sw the water saturation, Pc the capillary pressure and Df the fractal dimension.

### RESULTS AND DISCUSSION

Based on field observation the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units as described in Figure1. These units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their attained results of

the seismo diffusion

coefficient fractal dimension and capillary pressure fractal dimension are shown in Table 1. Based on the achieved results it was found that the seismo diffusion coefficient fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 allocated to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as shown in Table1. The seismo diffusion coefficient fractal dimension and capillary pressure fractal dimension were detected to increase with increasing permeability as proofed in Table1 owing to the possibility of having interconnected channels.

AGE	Fm.	Mbr.	unit	LITHO-LOGY	DESCRIPTION	
Late Permian	Khuff Formation	Huqayf Member			Limestone : Cream, dense, burrowed, thickness 6.56'	
					Sub-Khuff unconformity.	
Late Carboniferous - Permian	Shajara Formation	Upper Shajara Member	Upper Shajara mudstone		Mudstone : Yellow, thickness 17.7'	
			Upper Shajara Reservoir	SJ13▲	Sandstone : Light brown, cross-bedded, coarse-grained, poorly sorted, porous, friable, thickness 6.5'	
				SJ12▲	Sandstone : Yellow, medium-grained, very coarse-grained, poorly, moderately sorted, porous, friable, thickness 13.1'	
				SJ11▲		
			Middle Shajara Member	Middle Shajara mudstone		Mudstone : Yellow-green, thickness 11.8'
						Mudstone : Yellow, thickness 1.3'
					Mudstone : Brown, thickness 4.5'	
		Middle Shajara Reservoir	SJ10▲	Sandstone : Light brown, medium-grained, moderately sorted, porous, friable, thickness 3.6'		
			SJ9▲	Sandstone : Yellow, medium-grained, moderately well sorted, porous, friable, thickness 0.9'		
			SJ8▲			
			SJ7▲	Sandstone : Red, coarse-grained, medium-grained, moderately well sorted, porous, friable, thickness 13.4'		
		Lower Shajara Member	Lower Shajara Reservoir	SJ6▲	Sandstone : White with yellow spots, fine-grained, hard, thickness 2.6'	
				SJ5▲	Sandstone : Limonite, thickness 1.3'	
SJ4▲	Sandstone : White, coarse-grained, very poorly sorted, thickness 4.5'					
			Sandstone : White-pink, poorly sorted, thickness 1.6'			
			Sandstone : Yellow, medium-grained, well sorted, porous, friable, thickness 3.9'			
			Sandstone : Red, medium-grained, moderately well sorted, porous, friable, thickness 11.8'			
Early Devonian	Tawil Formation				Sub-Unayzah unconformity. Sandstone : White, fine-grained.	

Figure1. Surface type section of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation at latitude 26° 52' 17.4" longitude 43° 36' 18"

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The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were carefully chosen for capillary pressure measurement as proven in Table1. Their positive slopes of the first procedure log of the seismo diffusion coefficient to maximum seismo diffusion coefficient versus log wetting phase saturation (Sw) and negative slopes of the second procedure log capillary pressure (Pc) versus log wetting phase saturation (Sw) are clarified in Figure 2, Figure 3, Figure 4, Figure 5 and Table 1. Their seismo diffusion coefficient fractal dimension and capillary pressure fractal dimension values are revealed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was described from 1955 md to 56 md which reflects decrease in seismo diffusion coefficient fractal dimension from 2.7748 to 2.4379 as quantified in table 1. Again, an increase in grain size and permeability was proved from sample SJ4 whose seismo diffusion coefficient fractal dimension and capillary pressure fractal dimension was found to be 2.6843 as described in Table 1.

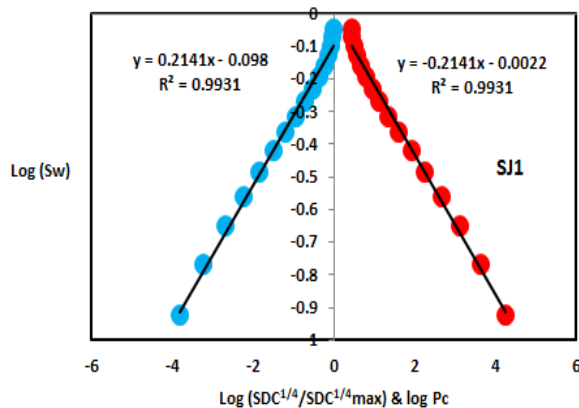


Figure2. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ1

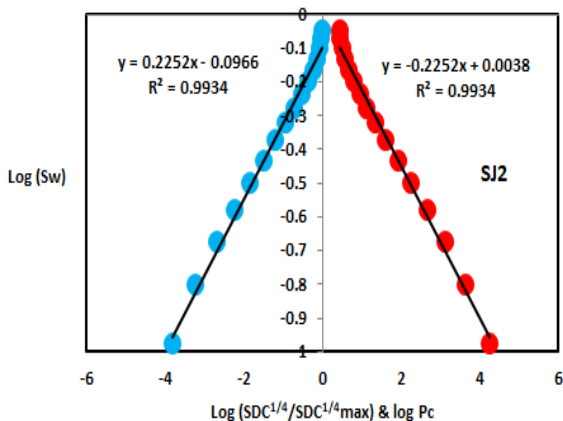


Figure3. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ2

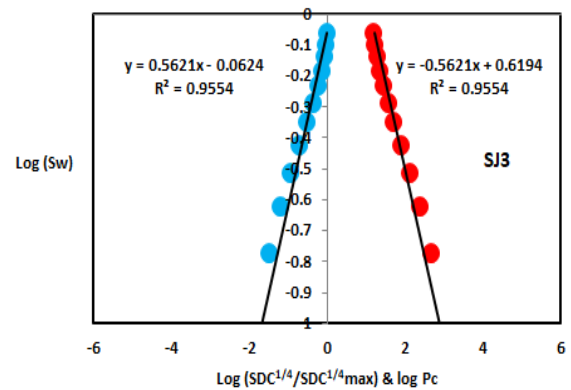


Figure4. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ3

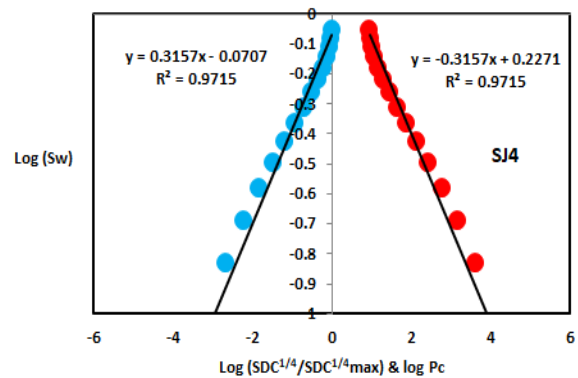


Figure5. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ4

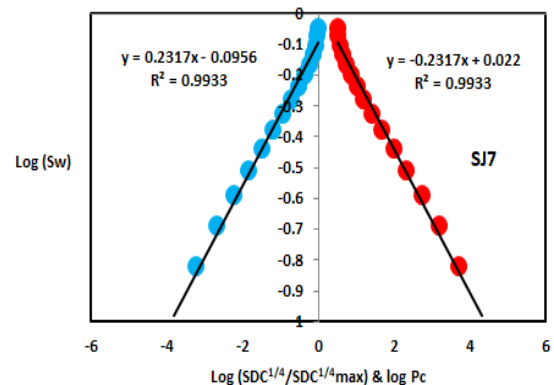


Figure6. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ7

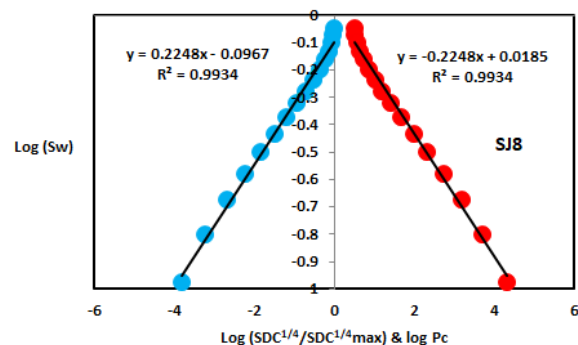
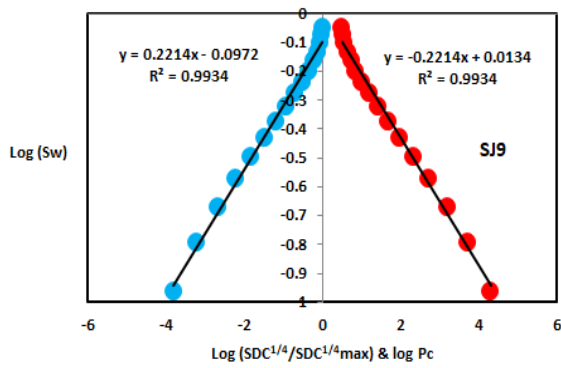
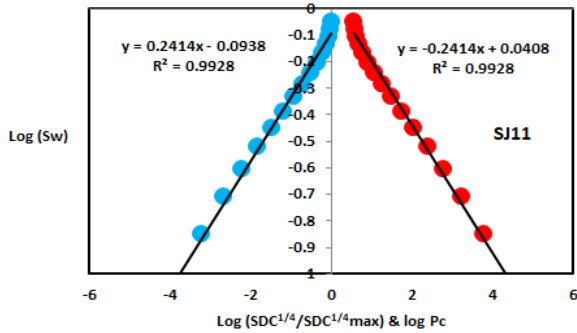


Figure7. Log (SDC1/4/SDC1/4max) & log pc versus log Sw for sample SJ8

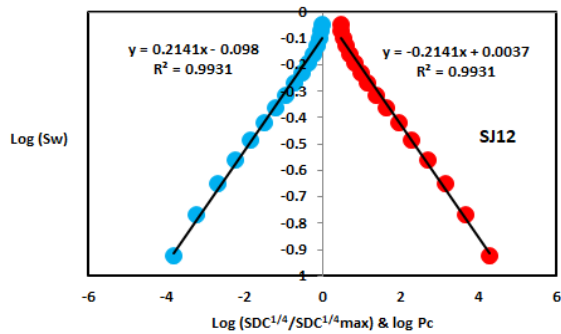
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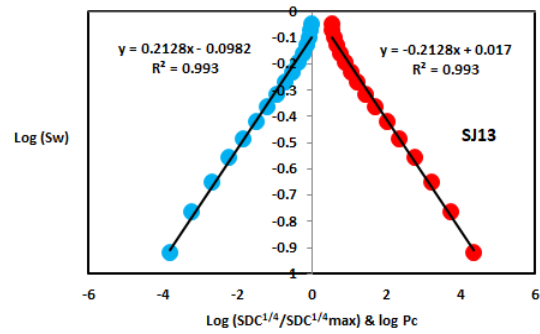
**Figure8.**  $\text{Log} (SDC1/4/SDC1/4max)$  &  $\text{log pc}$  versus  $\text{log Sw}$  for sample SJ9



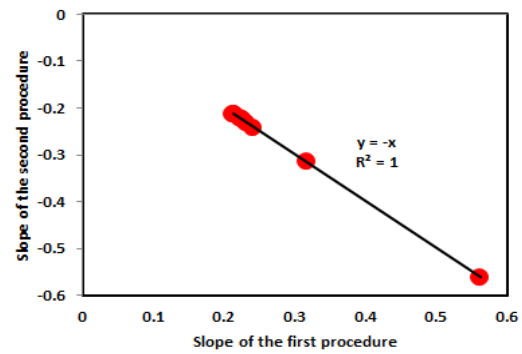
**Figure9.**  $\text{Log} (SDC1/4/SDC1/4max)$  &  $\text{log pc}$  versus  $\text{log Sw}$  for sample SJ11



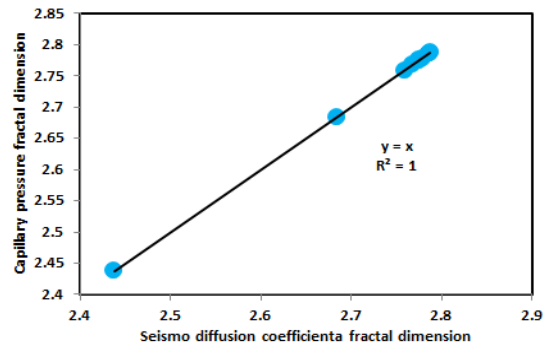
**Figure10.**  $\text{Log} (SDC1/4/SDC1/4max)$  &  $\text{log pc}$  versus  $\text{log Sw}$  for sample SJ12



**Figure11.**  $\text{Log} (SDC1/4/SDC1/4max)$  &  $\text{log pc}$  versus  $\text{log Sw}$  for sample SJ13



**Figure12.** Slope of the first procedure versus slope of the second procedure



**Figure13.** Seismo diffusion coefficient fractal dimension versus capillary pressure fractal dimension

**Table1.** Petro physical model showing the three Shajara Reservoir Units with their corresponding values of Seismo diffusion coefficient fractal dimension and capillary pressure fractal dimension

Formation	Reservoir	Sample	Porosity %	k(md)	Positive slope of the first procedure Slope=3-Df	Negative slope of the second procedure Slope=Df-3	Seismo diffusion coefficient fractal dimension	Capillary pressure fractal dimension
Permo-Carboniferous Shajara Formation	Upper Shajara Reservoir	SJ13	25	973	0.2128	-0.2128	2.7872	2.7872
		SJ12	28	1440	0.2141	-0.2141	2.7859	2.7859
		SJ11	36	1197	0.2414	-0.2414	2.7586	2.7586
	Middle Shajara Reservoir	SJ9	31	1394	0.2214	-0.2214	2.7786	2.7786
		SJ8	32	1344	0.2248	-0.2248	2.7752	2.7752
		SJ7	35	1472	0.2317	-0.2317	2.7683	2.7683
	Lower Shajara Reservoir	SJ4	30	176	0.3157	-0.3157	2.6843	2.6843
		SJ3	34	56	0.5621	-0.5621	2.4379	2.4379
		SJ2	35	1955	0.2252	-0.2252	2.7748	2.7748
		SJ1	29	1680	0.2141	-0.2141	2.7859	2.7859



In contrast, the Middle Shajara Reservoir which is separated from the Lower Shajara Reservoir by an unconformity surface as revealed in Figure 1. It was nominated by four samples (Figure 1), three of which named as SJ7, SJ8, and SJ9 as illuminated in Table 1 were chosen for capillary measurements as described in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are shown in Figure 6, Figure 7 and Figure 8 and Table 1. Furthermore, their seismo diffusion coefficient fractal dimensions and capillary pressure fractal dimensions show similarities as defined in Table 1. Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in table 1.

On the other hand, the Upper Shajara reservoir was separated from the Middle Shajara Reservoir by yellow green mudstone as shown in Figure 1. It is defined by three samples so called SJ11, SJ12, SJ13 as explained in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in Figure 9, Figure 10 and Figure 11 and Table 1. Moreover, their seismo diffusion coefficient fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as simplified in table 1.

Overall a plot of positive slope of the first procedure versus negative slope of the second procedure as described in Figure 12 reveals three permeable zones of varying Petro physical properties. These reservoir zones were also confirmed by plotting seismo diffusion coefficient fractal dimension versus capillary pressure fractal dimension as described in Figure 13. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment.

## CONCLUSION

The sandstones of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units based on seismo diffusion coefficient fractal dimension. The Units from base to top are: Lower Shajara Seismo diffusion coefficient Fractal Dimension Unit, Middle Shajara Seismo diffusion coefficient Fractal Dimension Unit, and Upper Shajara Seismo diffusion coefficient Fractal Dimension Unit. These units were also proved

by capillary pressure fractal dimension. The fractal dimension was found to increase with increasing grain size and permeability owing to possibility of having interconnected channels.

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## REFERENCES

- [1] Frenkel J. On the theory of seismic and seismoelectric phenomena in a moist soil. *Journal of Physics*. 1944;3(4):230-241.
- [2] Li K and Williams W Determination of capillary pressure function from resistivity data. *Transport in Porous Media*. 2007; 67(1):1-15.
- [3] A Revil and A Jardani Seismoelectric response of heavy oil reservoirs: theory and numerical modelling. *Geophysical Journal International*. 2010; 180(2):781-797.
- [4] Dukhin A, Goetz Pand Thommes M. Seismoelectric effect: a non-isochoric streaming current. 1 Experiment. *Journal of Colloid and Interface Science*. 2010;345(2):547-553.
- [5] Guan W, Hu H and Wang Z. Permeability inversion from low-frequency seismoelectric logs in fluid- saturated porous formations, *Geophysical Prospecting*. 2012;61(1):120-133.
- [6] Hu, H., Guan, W., and Zhao, W. Theoretical studies of permeability inversion from seismoelectric logs. *Geophysical Research Abstract*. 2012;14: EGU2012-6725-1 2012 EGU General Assembly 2012.
- [7] Borde C, S'én'échal P, Barri`ere J and et al. Impact of water saturation on seismoelectric transfer functions: a laboratory study of co-seismic phenomenon, *Geophysical Journal International* 2015;200(3):1317-1335.
- [8] Jardani A, and Revil A. Seismoelectric couplings in a poroelastic material containing two immiscible fluid phases, *Geophysical Journal International* 2015; 202(2): 850-870.
- [9] Holzhauser J, Brito D, Bordes C and et al. Experimental quantification of the seismoelectric transfer function and its dependence on conductivity and saturation in loose sand, *Geophysical Prospecting* 2016 ;65(4):1097-1120.
- [10] Ping R, Wei J-X, Di B-R and et al. Experimental

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- research on seismoelectric effects in sandstone. *Applied Geophysics* 2016; 13(3):425-436.
- [11] U Djuraev, S.R. Jufar, and P Vasant. Numerical Study of frequency-dependent seismo electric coupling in partially-saturated porous media. *MATEC Web of Conferences* 87,02001(2017).
- [12] Alkhidir KEME. Pressure head fractal dimension for characterizing Shajara Reservoirs of the Shajara Formation of the Permo-Carboniferous Unayzah Group, Saudi Arabia. *Archives of Petroleum and Environmental and Biotechnology* 2017;2:1-7.
- [13] Al-Khidir KE. On Similarity of Pressure Head and Bubble Pressure Fractal Dimensions for Characterizing Permo-Carboniferous Shajara Formation, Saudi Arabia. *Journal of Industrial Pollution and Toxicity*.2018; 1(1):1-10.
- [14] Alkhidir KEME. Geometric relaxation time of induced polarization fractal dimension for characterizing Shajara Reservoirs of the Shajara Formation of the Permo-Carboniferous Unayzah Group, Saudi Arabia, *Sci Fed Journal of Petroleum*.2018;2(1):1-6.
- [15] Alkhidir KEME. Geometric relaxation time of induced polarization fractal dimension for characterizing Shajara Reservoirs of the Shajara formation of the Permo-Carboniferous Unayzah Group-Permo. *International Journal of Petro chemistry and Research*. 2018;2(1):105-108.

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