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On Similarity of Molar heat Capacity and Capillary Pressure Fractal Dimensions for Characterizing Shajara Reservoirs of the Permo-Carboniferous Shajara Formation, Saudi Arabia

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ABSTRACT

The quality and assessment of a reservoir can be documented in details by the application of Molar heat capacity. This research aims to calculate fractal dimension from the relationship among Molar heat capacity, maximum Molar heat capacity and wetting phase saturation and to approve it by the fractal dimension derived from the relationship among capillary pressure and wetting phase saturation. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, Molar heat capacity, maximum Molar heat capacity and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between Molar heat capacity and maximum Molar heat capacity versus logarithm wetting phase saturation. The slope of the first procedure = 3- Df (fractal dimension). The second procedure for obtaining the fractal dimension was determined by plotting the logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure = Df -3. On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units.

Keywords: *Shajara Reservoirs; Shajara Formation; Molar heat capacity fractal dimension; Capillary pressure fractal dimension*

INTRODUCTION

Seismo electric effects related to electro kinetic potential, dielectric permittivity, pressure gradient, fluid viscosity, and electric conductivity was first reported by [1]. Capillary pressure follows the scaling law at low wetting phase saturation was reported by [2]. Seismo electric phenomenon by considering electro kinetic coupling coefficient as a function of effective charge density, permeability, fluid viscosity and electric conductivity was reported by [3].

The magnitude of seismo electric current depends on porosity, pore size, zeta potential of the pore surfaces, and elastic properties of the matrix was investigated by [4]. The tangent of the ratio of converted electric field to pressure is approximately in inverse proportion to permeability was studied by [5]. Permeability inversion from seismoelectric log at low frequency was studied by [6]. They reported that, the tangent of the ratio among electric excitation intensity and pressure field is a function of porosity, fluid viscosity, frequency, tortuosity, and fluid density and Dracy permeability. A decrease of seismoelectric frequencies with increasing water content was reported by [7]. An increase of seismo electric transfer function with increasing water saturation was studied by [8]. An increase of dynamic seismo electric transfer function with decreasing fluid conductivity was described by[9]. The amplitude of seismo electric signal increases with increasing permeability which means that the seismo electric effects are directly related to the permeability and can be used to study the permeability of the reservoir was illustrated by [10]. Seismo electric coupling is frequency dependent and decreases expontialy when frequency increases was demonstrated by [11]. An increase of permeability with increasing pressure head and bubble pressure fractal dimension was reported by [12, 13]. An increase of geometric relaxation time of induced polarization fractal dimension with permeability increasing and grain size was described by [14, 15].

Materials and Methods

Sandstone samples were collected from the surface type section of the Permo-Carboniferous Shajara Formation, latitude 26° 52' 17.4", longitude 43° 36' 18". (Figure1). Porosity was measured on collected samples using mercury intrusion Porosimetry and permeability was derived from capillary pressure data. The purpose of this paper is to obtain Molar heat capacity fractal dimension and to confirm it by capillary pressure fractal dimension. The fractal dimension of the first procedure is determined from the positive slope of the plot of logarithm of the ratio of Molar heat capacity to maximum Molar heat capacity log ($MHC^{1/2}/MHC^{1/2}_{max}$) versus log wetting phase saturation (logSw). Whereas the fractal dimension of the second procedure is determined from the negative slope of the plot of logarithm of capillary pressure (log Pc) versus logarithm of wetting phase saturation (log Sw).

The molar heat capacity can be scaled as

$$\mathbf{S_w = \left[\frac{MHC^{\frac{1}{2}}}{MHC^{\frac{1}{2}}_{max}}\right]^{[3-Df]}} \qquad 1$$

Where Sw the water saturation, MHC the molar heat capacity in Joule / kelvin * mole, MHC_{max} the maximum molar heat capacity in Joule / kelvin* kilo gram, and Df the fractal dimension.

Equation 1 can be proofed from

$$\mathbf{Q = \Delta T * NOM * MHC} \qquad 2$$

Where Q the heat in Joule, ΔT temperature difference in kelvin, NOM the number of moles, MHC the molar heat capacity in Joule / kelvin * mole.

The NOM the number of moles, can be scaled as

$$\mathbf{NOM = \left[\frac{m}{mm}\right]} \qquad 3$$

Where NOM the number of moles, m the mass in kilo gram, mm the molar mass in kilo gram / mole

Insert equation 3 into equation 2

$$\mathbf{Q = \left[\frac{\Delta T * m * MHC}{mm}\right]} \qquad 4$$

The mass m can be scaled as

$$\mathbf{m = \left[\frac{F}{g}\right]} \qquad 5$$

Where m the mass in kilo gram, F the force in Newton, g acceleration in meter / square second

Insert equation 5into equation 4

$$\mathbf{Q = \left[\frac{\Delta T * F * MHC}{mm * g}\right]} \qquad 6$$

The acceleration g can be scaled as

$$\mathbf{g = \left[\frac{E}{\psi}\right]} \qquad 7$$

Where g the acceleration in meter / square second, E the electric field in volt / meter, ψ the electric transfer function in volt * square second / square meter.

Insert equation 7 into equation 6

$$\mathbf{Q = \left[\frac{\Delta T * F * MHC * \psi}{mm * E}\right]} \qquad 8$$

The electric field E can be scaled as

$$\mathbf{E = \left[\frac{V}{CEK}\right]} \qquad 9$$

Where E the electric field in volt / meter, V the velocity in meter / second, CEK the electro kinetic coefficient in ampere / pascal * meter

Insert equation 9 into equation 8

$$\mathbf{Q = \left[\frac{\Delta T * F * MHC * \psi * CEK}{mm * V}\right]} \qquad 10$$

The velocity V can be scaled as

$$\mathbf{V = \left[\frac{Q'}{A}\right]} \qquad 11$$

Where V the velocity in meter / second, Q' the flow rate in cubic meter / second, A the area in square meter

Insert equation 11 into equation 10

$$\mathbf{Q = \left[\frac{\Delta T * F * MHC * \psi * CEK * A}{mm * Q'}\right]} \qquad 12$$

Equation 10 after rearrange will become

$$\mathbf{Q * Q' * mm = \Delta T * F * MHC * \psi * CEK * A} \qquad 13$$

The flow rate Q' can be scaled as

$$\mathbf{Q' = \left[\frac{k * A * \Delta P}{\mu * L}\right]} \qquad 14$$

Where Q' the flow rate in cubic meter / second, k the permeability in square meter, A the area in square meter, ΔP the differential pressure in pascal, μ the fluid viscosity in pascal second, L the capillary length in meter.

Insert equation 14 into equation 13

$$\mathbf{Q * k * A * \Delta P * mm = \Delta T * F * MHC * \psi * CEK * A * \mu * L} \quad \mathbf{15}$$

The maximum permeability k $_{max}$ can be scaled as

$$\mathbf{Q * k_{max} * A * \Delta P * mm = \Delta T * F * MHC_{max} * \psi * CEK * A * \mu * L} \quad \mathbf{16}$$

Divide equation 15 by equation 16

$$\left[\frac{\mathbf{Q * k * A * \Delta P * mm}}{\mathbf{Q * k_{max} * A * \Delta P * mm}}\right] = \left[\frac{\mathbf{T * F * MHC * \psi * CEK * A * \mu * L}}{\mathbf{\Delta T * F * MHC_{max} * \psi * CEK * A * \mu * L}}\right] \quad \mathbf{17}$$

Equation 17 after simplification will become

$$\left[\frac{\mathbf{k}}{\mathbf{k_{max}}}\right] = \left[\frac{\mathbf{MHC}}{\mathbf{MHC_{max}}}\right] \quad \mathbf{18}$$

Take the square root of equation 18

$$\sqrt{\left[\frac{\mathbf{k}}{\mathbf{k_{max}}}\right]} = \sqrt{\left[\frac{\mathbf{MHC}}{\mathbf{MHC_{max}}}\right]} \quad \mathbf{19}$$

Equation 19 after simplification will become

$$\left[\frac{\mathbf{k^{\frac{1}{2}}}}{\mathbf{k_{max}^{\frac{1}{2}}}}\right] = \left[\frac{\mathbf{MHC^{\frac{1}{2}}}}{\mathbf{MHC_{max}^{\frac{1}{2}}}}\right] \quad \mathbf{20}$$

$$\mathbf{But;}\ \left[\frac{\mathbf{k^{\frac{1}{2}}}}{\mathbf{k_{max}^{\frac{1}{2}}}}\right] = \left[\frac{\mathbf{MHC^{\frac{1}{2}}}}{\mathbf{MHC_{max}^{\frac{1}{2}}}}\right] \quad \mathbf{21}$$

Where r the pore radius in meter, r $_{max}$ the maximum pore radius in meter

Take the logarithm of equation 21

$$\mathbf{log}\left[\frac{\mathbf{k^{\frac{1}{2}}}}{\mathbf{k_{max}^{\frac{1}{2}}}}\right] = \mathbf{log}\left[\frac{\mathbf{MHC^{\frac{1}{2}}}}{\mathbf{MHC_{max}^{\frac{1}{2}}}}\right] = \mathbf{log}\left[\frac{\mathbf{r}}{\mathbf{r_{max}}}\right] \quad \mathbf{22}$$

$$\mathbf{But;\ log}\left[\frac{\mathbf{r}}{\mathbf{r_{max}}}\right] = \left[\frac{\mathbf{log\, S_w}}{\mathbf{3 - Df}}\right] \quad \mathbf{23}$$

Insert equation 23 into equation 22

$$\mathbf{log}\left[\frac{\mathbf{MHC^{\frac{1}{2}}}}{\mathbf{MHC_{max}^{\frac{1}{2}}}}\right] = \left[\frac{\mathbf{log\, S_w}}{\mathbf{3 - Df}}\right] \quad \mathbf{24}$$

Equation 24 after log removal will become

$$\mathbf{S_w} = \left[\frac{\mathbf{MHC^{\frac{1}{2}}}}{\mathbf{MHC_{max}^{\frac{1}{2}}}}\right]^{\mathbf{[3-Df]}} \quad \mathbf{25}$$

Equation 25 the proof of equation 1 which relates the water saturation, molar heat capacity, maximum molar heat capacity and the fractal dimension.

The capillary pressure can be scaled as

$$\mathbf{Sw = [Df - 3] * Pc * constant} \quad \mathbf{26}$$

Where Sw the water saturation, Pc the capillary pressure and Df the fractal dimension.

Results and Discussion

Based on field observation the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units as described in Figure1.These units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their attained results of the Molar heat capacity fractal dimension and capillary pressure fractal dimension are shown in Table 1. Based on the achieved results it was found that the Molar heat capacity fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 allocated to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as shown in Table1. The Molar heat capacity fractal dimension and capillary pressure fractal dimension were detected to increase with increasing permeability as proofed in Table1 owing to the possibility of having interconnected channels.

The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were carefully chosen for capillary pressure measurement as proven in Table1. Their positive slopes of the first procedure log of the Molar heat capacity to maximum Molar heat capacity versus log wetting phase saturation (Sw) and negative slopes of the second procedure log capillary pressure (Pc) versus log wetting phase saturation (Sw) are clarified in Figure 2, Figure 3, Figure 4, Figure 5 and Table 1.

Their Molar heat capacity fractal dimension and capillary pressure fractal dimension values are revealed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was described from 1955 md to 56 md which reflects decrease in Molar heat capacity fractal dimension from

2.7748 to 2.4379 as quantified in table1.Again, an increase in grain size and permeability was proved from sample SJ4 whose Molar heat capacity fractal dimension and capillary pressure fractal dimension was found to be 2.6843 as described in Table 1.

Table1. *Petro physical model showing the three Shajara Reservoir Units with their corresponding values of Molar heat capacity fractal dimension and capillary pressure fractal dimension*

Formation	Reservoir	Sample	Porosity %	k (md)	Positive slope of the first procedure Slope=3-Df	Negative slope of the second procedure Slope=Df-3	Molar heat capacity fractal dimension	Capillary pressure fractal dimension
Permo-Carboniferous Shajara Formation	Upper Shajara Reservoir	SJ13	25	973	0.2128	-0.2128	2.7872	2.7872
		SJ12	28	1440	0.2141	-0.2141	2.7859	2.7859
		SJ11	36	1197	0.2414	-0.2414	2.7586	2.7586
	Middle Shajara Reservoir	SJ9	31	1394	0.2214	-0.2214	2.7786	2.7786
		SJ8	32	1344	0.2248	-0.2248	2.7752	2.7752
		SJ7	35	1472	0.2317	-0.2317	2.7683	2.7683
	Lower Shajara Reservoir	SJ4	30	176	0.3157	-0.3157	2.6843	2.6843
		SJ3	34	56	0.5621	-0.5621	2.4379	2.4379
		SJ2	35	1955	0.2252	-0.2252	2.7748	2.7748
		SJ1	29	1680	0.2141	-0.2141	2.7859	2.7859

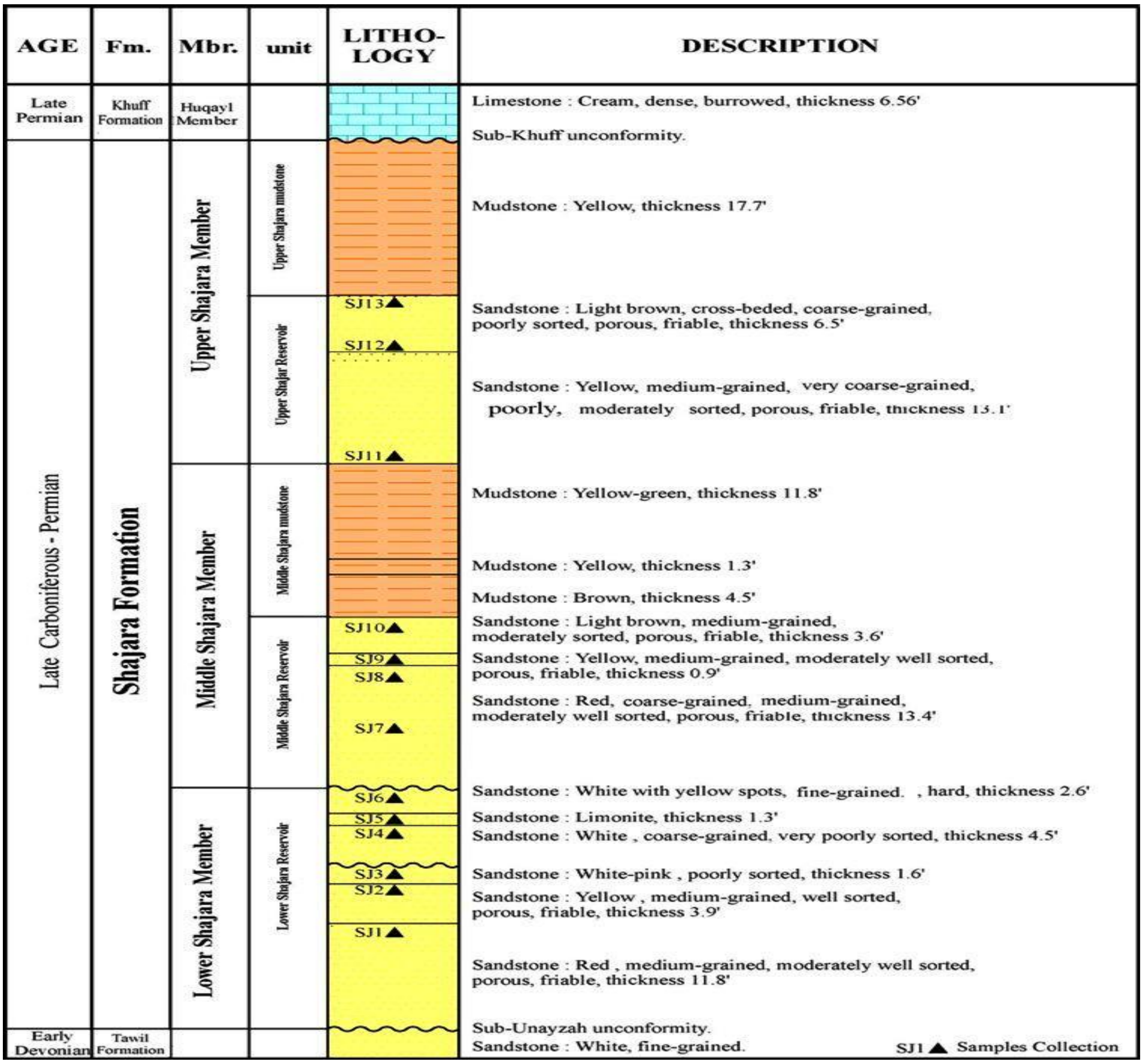

Figure1. *Surface type section of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation at latitude 26° 52' 17.4'' longitude 43° 36' 18''*

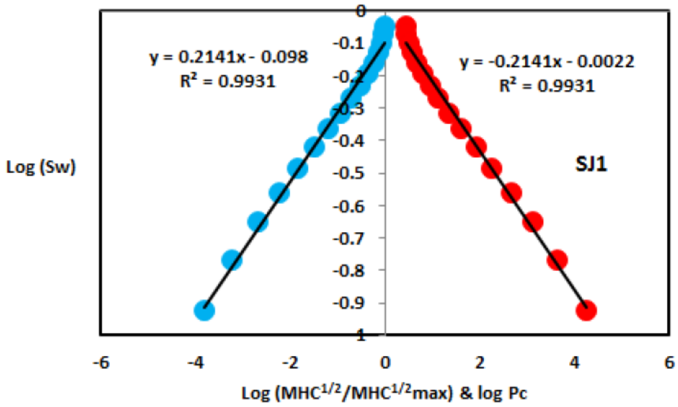

Figure2. *Log ($MHC^{1/2}$/$MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ1*

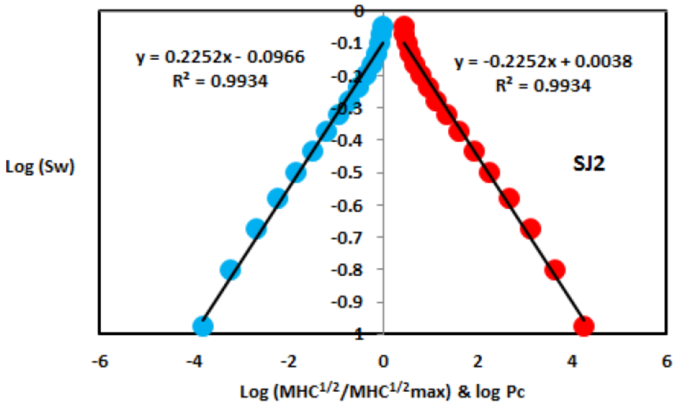

Figure3. *Log ($MHC^{1/2}$/$MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ2*

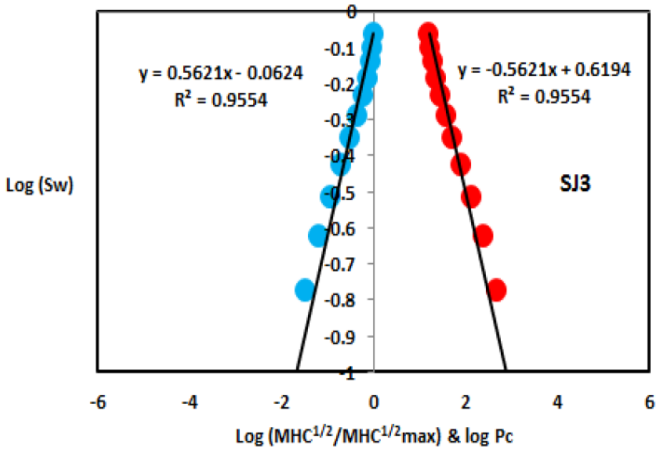

Figure4. *Log ($MHC^{1/2}$/$MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ3*

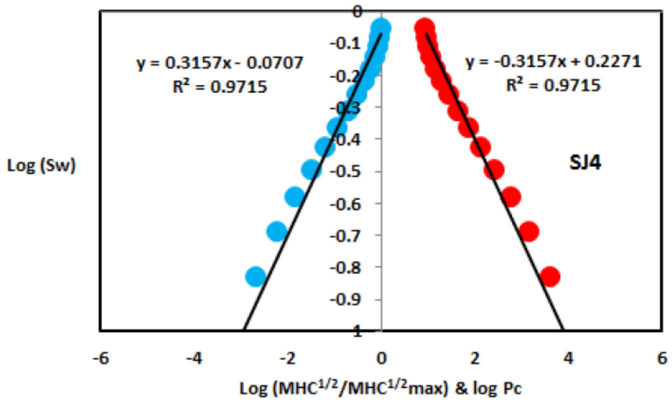

Figure5. *Log ($MHC^{1/2}$/$MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ4*

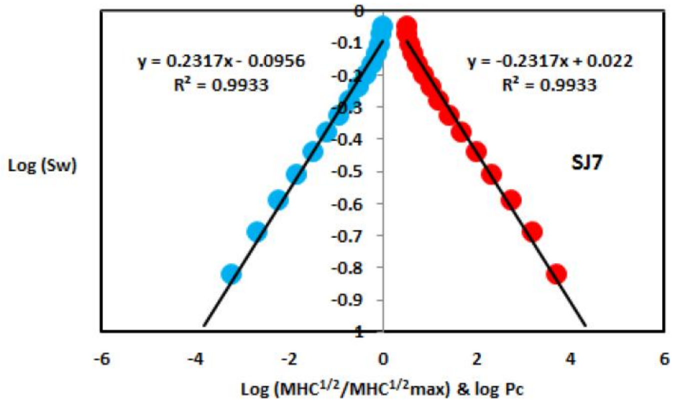

Figure6. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ7*

In contrast, the Middle Shajara reservoir which is separated from the Lower Shajara reservoir by an unconformity surface as revealed in Figure 1. It was nominated by four samples (Figure 1), three of which named as SJ7, SJ8, and SJ9 as illuminated in Table1 were chosen for capillary measurements as described in Table 1.Their positive slopes of the first procedure and negative slopes of the second procedure are shown in Figure 6, Figure 7 and Figure 8 and Table 1. Furthermore, their Molar heat capacity fractal dimensions and capillary pressure fractal dimensions show similarities as defined in Table 1. Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in table 1.

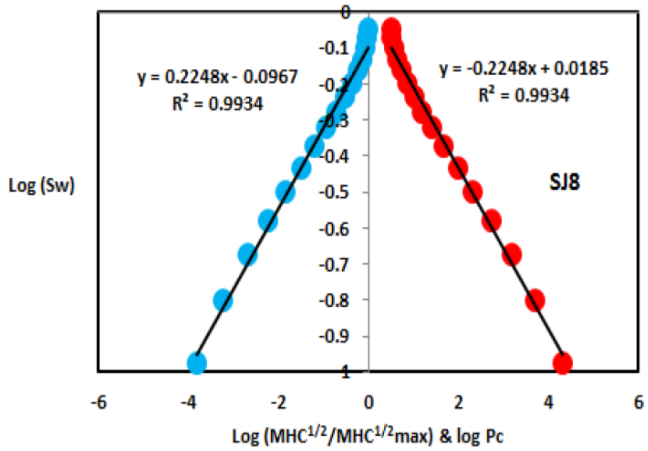

Figure7. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ8*

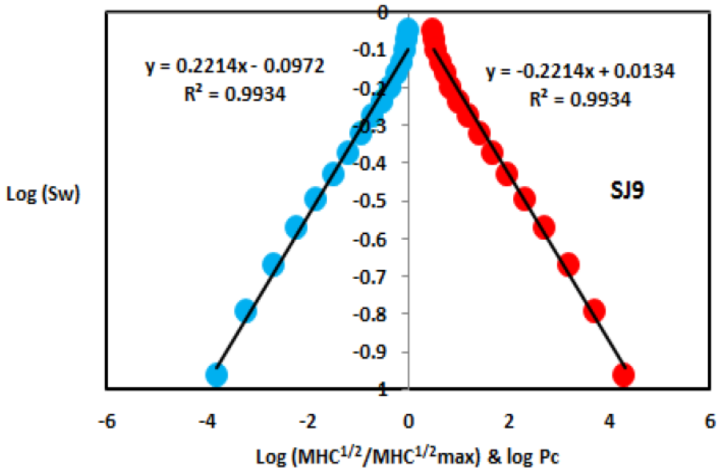

Figure8. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ9*

On the other hand, the Upper Shajara reservoir was separated from the Middle Shajara reservoir by yellow green mudstone as shown in Figure 1. It is defined by three samples so called SJ11, SJ12, and SJ13 as explained in Table1. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in Figure 9, Figure 10 and Figure 11 and Table1. Moreover, their Molar heat capacity fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as simplified in table 1. Overall a plot of positive slope of the first procedure versus negative slope of the second procedure as described in Figure 12 reveals three permeable zones of varying Petro physical properties. These reservoir zones were also confirmed by plotting Molar heat capacity fractal dimension versus capillary pressure fractal dimension as described in Figure 13. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment.

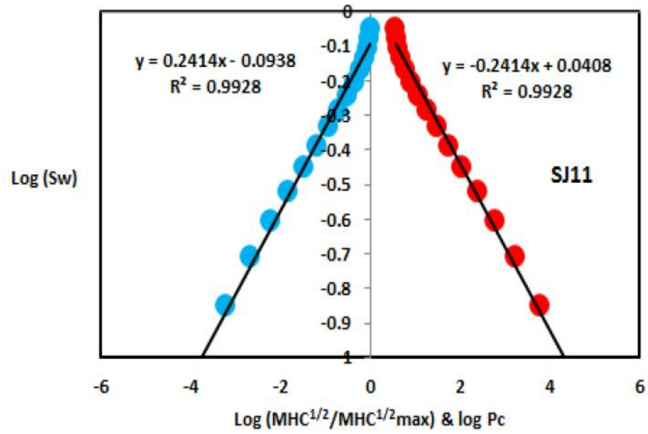

Figure9. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ11*

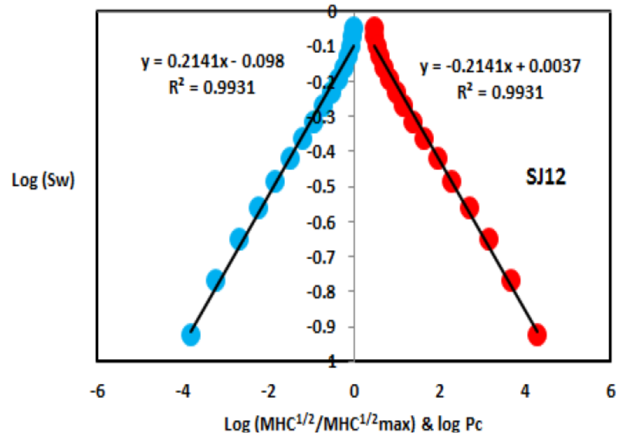

Figure10. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sampleSJ12*

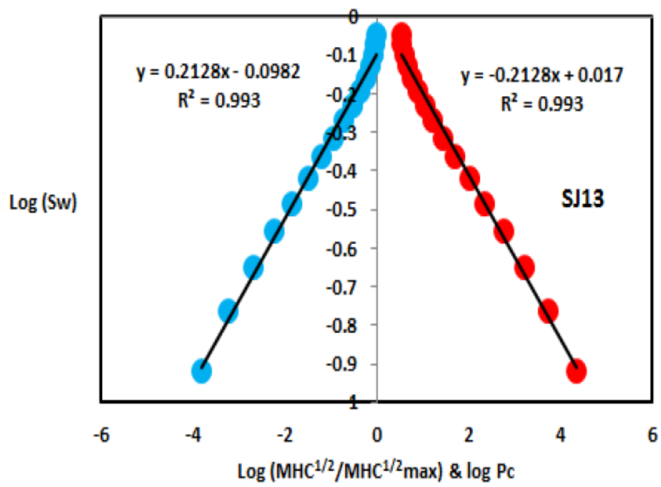

Figure11. *Log ($MHC^{1/2}/MHC^{1/2}_{max}$) & log pc versus log Sw for sample SJ13*

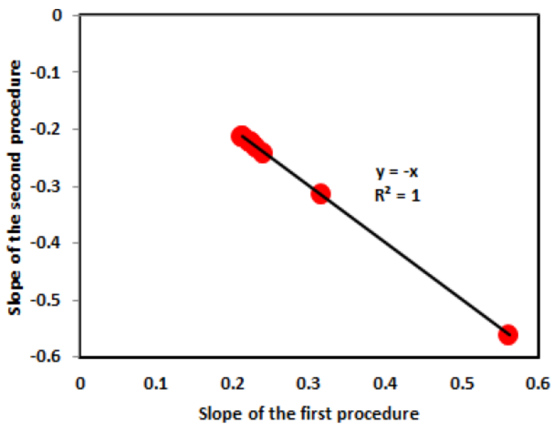

Figure12. *Slope of the first procedure versus slope of the second procedure*

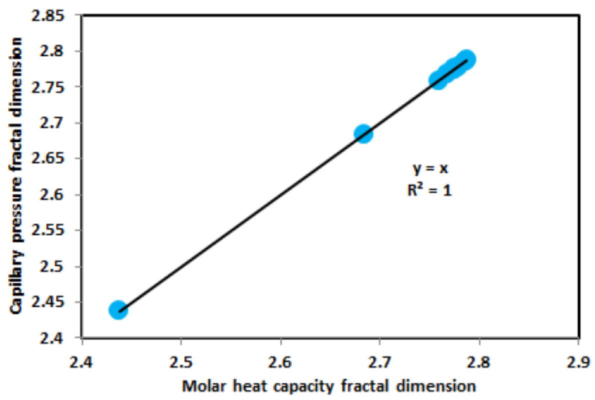

Figure13. *Molar heat capacity fractal dimension versus capillary pressure fractal dimension*

Conclusion

The sandstones of the Shajara Reservoirs of the permo-Carboniferous Shajara Formation were divided here into three units based on Molar heat capacity fractal dimension. The Units from base to top are: Lower Shajara Molar Heat Capacity Fractal Dimension Unit, Middle Shajara Molar Heat Capacity Fractal Dimension Unit, and Upper Shajara Molar Heat Capacity Fractal Dimension Unit. These units were also proved by capillary pressure fractal dimension. The fractal dimension was found to increase with increasing grain size and permeability owing to possibility of having interconnected channels.

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